The Differential Effects of New Privacy Protections on Publisher and Advertiser Profitability

Priyanka Sharma, Yidan Sun, and Liad Wagman

February 2021

DATA Catalyst

Abstract

We model an asymmetric ad-network duopoly with heterogeneous publishers and advertisers to assess the effects of data and tracking restrictions on profits. Publishers and advertisers vary in their sizes and preferences towards the two ad networks, and networks differ in terms of the revenues they can deliver to publishers and advertisers. We show that new restrictions that hamper networks' ability to deliver revenues lead to the highest percentage profit declines for small publishers and small advertisers, particularly those who contract with the lower-revenue ad network. We demonstrate that our findings are robust to regulations that differentially impact ad networks and to other market settings.

Keywords: Ad networks, publishers, advertisers, data, privacy, tracking

JEL Classifications: M37, M38, L24

*Wagman and Sun gratefully acknowledge support from Data Catalyst (https://www.datacatalyst.org/).

†Stuart School of Business, Illinois Institute of Technology. Email: priyanka.sharma@stuart.iit.edu.
‡Stuart School of Business, Illinois Institute of Technology. Email: ysun100@hawk.iit.edu.
§Stuart School of Business, Illinois Institute of Technology. Email: lwagman@stuart.iit.edu.

1 Introduction

Online content publishers partner with sell-side ad networks in order to monetize their content. As part of the partnership, a publisher may insert a piece of code from their chosen ad network into their website or mobile app or choose a network on which to deliver content directly. Doing so enables publishers to sell virtual ad space. Buy-side ad networks can then bring in advertisers who compete to advertise to the different audiences of publishers' content, and the revenue from the advertisers is shared among the networks and the publishers. Some networks offer both sell-side and buy-side services. Networks may use consumer data to better target ads to users browsing publishers' content, where a network's targeting ability is influenced by the data it has, the number of advertisers and/or publishers who participate in its network, the type of content publishers offer, the type of user traffic the publishers' content attracts, and the regulatory environment governing data utilization, among other factors. Publisher-network-advertiser relationships are ubiquitous, and include ad monetization on publishers' websites (e.g., through Google AdSense, Verizon Media, Taboola), mobile apps (e.g. Facebook Advertising Network, Google AdMob), and streamed content (e.g., YouTube, Twitch, Facebook).

The use of consumer data by ad networks for targeted marketing has heightened privacy concerns, leading governments worldwide to consider laws that enhance data protection by restricting its use, imposing consent or legitimate interest requirements for its collection, and limiting firms' ability to track consumers' activities (Acquisti et al., 2016). A rise in data protection can reduce the ability of ad networks to efficiently match ads with users, which may be one of the intended effects, but the downstream consequences for publishers and advertisers have not been entirely clear.[1] This paper examines differences in the downstream effects on publishers and advertisers as a result of a change in how much revenue ad networks can deliver (driven, for instance, by data regulation, or by private sector efforts such as Intelligent Tracking Prevention[2]), as a function of their size and which ad network they utilize.

Our model incorporates an asymmetric duopoly (Thisse and Vives, 1988) ad-network mar-

[1]A recent study demonstrated that publishers gain additional revenue when able to utilize more cookies on users' browsers (Marotta et al., 2019). On the other hand, a regulatory regime that requires a publisher to direct users to a version of its web property that does not utilize cookies forfeits the additional revenue from behavioral targeting.

[2]See, e.g., `https://webkit.org/blog/9521/intelligent-tracking-prevention-2-3/` and `https://www.ecommercetimes.com/story/86410.html`.

ket, to capture the fact that one network may have more user data than another, which may allow that network to attract more advertisers and bring in larger ad revenues for publishers. Each of the ad networks sets revenue-sharing agreements for publishers,[3] heterogeneous publishers select a network with which to contract, and heterogeneous advertisers choose a network on which to advertise. We distinguish between small and large publishers and small and large advertisers through their different abilities to attract and determine revenue-sharing and advertising agreements with the networks, respectively. Specifically, small publishers utilize an ad network's automated system,[4] where revenue-sharing terms tend to be uniform for participating publishers.[5] Large publishers, in contrast, can attract and negotiate individual agreements with the networks; that is, the networks compete over their business, potentially placing those publishers in a stronger bargaining position.[6] We analogously distinguish between small and large advertisers based on any discounts (e.g., per click, ad impression, minimum bids, or quality scores) the latter may obtain due to their larger scale. This distinction is motivated both anecdotally, as it appears to be common in the industry, as well as by the literature (see, e.g., Adachi and Tremblay, 2020).

Ad networks may maintain detailed information about users, often acquired through cookies that track and record users' web activities.[7] A stricter ruleset for utilizing data about users has been linked to reductions in user tracking (Libert et al., 2018), ad performance (Goldfarb and Tucker, 2011; Goldberg et al., 2019), consumer surplus (D'Annunzio and Russo, 2019), and other data-related monetization (Kim and Wagman, 2015; Johnson et al., 2020).[8] Given that ad networks may have different user data (e.g., through access to other business verticals, by having access to richer past ad performance data, by keeping users logged in via other web products,

[3] See, for instance, `https://support.google.com/adsense/answer/180195?hl=en`.

[4] E.g., `https://www.google.com/adsense/start/` and `https://www.facebook.com/audiencenetwork`.

[5] This tends to be particularly true for publishers of a similar size or revenue range. Some networks may offer different terms for, e.g., different revenue ranges.

[6] It has been reported that large publishers and advertisers, as well as smaller publishers who band together to form alliances, may be able to negotiate better revenue-sharing terms with networks. See, e.g., `https://bit.ly/2t70lsU`, `https://bit.ly/2LLzs4s`, `https://tcrn.ch/38tO8ie`, and `https://bit.ly/2PKvRou`. In Germany, for instance, four of the largest publishers banded together (`https://bit.ly/2rwwlXc`), with similar proposals elsewhere (see, e.g., the Journalism Competition and Preservation Act, `https://bit.ly/2PdO8eI`). Our distinction between small and large actors can be alternately, but less cleanly, modeled as heterogeneous bargaining powers in, e.g., a reduced-form Nash bargaining game with similar qualitative results.

[7] See, e.g., `https://on.wsj.com/2MamqxF`.

[8] See, also, `https://9to5mac.com/2019/12/09/apple-safari-privacy-feature-ads/` and `https://bit.ly/3coOj2I`.

and by being able to utilize more data sources through affiliations that a data protection law permits), the changes in their revenues when new data restrictions are imposed may differ.[9] In a simplistic sense, a regulation that enhances data protection can be viewed as affecting ad networks in a proportional way, with a higher magnitude impact on the network that brings in more ad revenues. Our baseline model considers this scenario as a benchmark. In practice, such a proportional impact is unlikely to be the case, with empirical evidence (Adjerid and de Matos, 2019; Jia et al., 2019; Johnson et al., 2019) and predictions (Campbell et al., 2015; Jin and Wagman, 2020) that point to the contrary. Ad networks may also collect and utilize data to different degrees, and network effects tend to be at play. We extend the base model to examine the impact of new data restrictions when the restrictions themselves asymmetrically impact the ad networks. Throughout, our focus is on assessing how these sort of changes may affect the profits of advertisers and publishers.

Policymakers are grappling with how to implement restrictions on data uses for marketing purposes. Some, like the European Union and the State of California, have already implemented new restrictions. However, there remains a lack of clarity about who stands to gain and who stands to lose, and to what extent, from restrictions that degrade the ability of ad networks to match users with ads, particularly in the downstream among publishers and advertisers.

Facing data utilization restrictions that hamper the two ad networks' abilities to generate revenues for publishers and advertisers in our model, our findings show that (i) smaller publishers and advertisers incur larger percentage profit declines, and (ii) the effects are more pronounced for those publishers and advertisers that contract with the weaker (in terms of ad revenues) of the two ad networks. The intuition for (i) is that smaller market actors, driven by worse outside options and/or lower revenues, have higher profit elasticities with respect to the restrictiveness of data protections, which makes them more vulnerable to stricter data rulesets. The intuition for (ii) is that proportional effects from a more restrictive data regime partially even out the playing field between the weaker and stronger ad networks, which forces the stronger network to make some concessions to its publishers and advertisers, dampening the negative effects on their profits. Consequently, in equilibrium, the publishers and advertisers working with the weaker network incur relatively higher profit declines.

[9] Moreover, a network may even gain after a rise in data protection. For instance, if advertisers shift their spending on one network to another, the latter may see a net rise in its ad performance and revenue.

Our contribution builds on works by Fudenberg and Tirole (2000), Cooper et al. (2005), Conitzer et al. (2012), Taylor and Wagman (2014), and Kim et al. (2019) who utilize related models to study the consumer-facing side of analogous markets. We extend the analyses in these related works by incorporating asymmetric ad networks and focusing specifically on the differential effects on publisher and advertiser profits. More specifically, these works use similar models to study the effects of stricter data rulesets on consumer surplus, and show that they depend on market structure and competition. Their analyses focus on sellers-consumers interactions, where sellers may track consumers' purchase histories or have some information from data brokers about consumers. Our analysis, in contrast, focuses on how online publishers, as sellers of virtual ad space, and advertisers, as potential buyers of that space, may be affected by stricter data rulesets that limit the ability of ad networks to bring them together.

Researchers have also focused on the potential pro-competitive or anti-competitive effects of access to user data and the corresponding implications for consumer surplus (De Cornière and Taylor, 2020), as well as on the relationship between targeted advertising and consumers' browsing activities (De Cornière, 2016), and the incentives of ad networks to disclose information about users and price this information to advertisers (Bergemann and Bonatti, 2015; De Cornière and De Nijs, 2016). Our focus in this paper is complementary, in the sense that understanding the differential consequences to stakeholder profits, in terms of the responsiveness or elasticities of those profits to changes in data availability, can be an important input to models that derive aggregate predictions (e.g., regarding advertiser or publisher profits). Moreover, our model extends prior analyses to a framework with asymmetric ad networks.

Related to our work, Kirpalani and Philippon (2020) examine a model where consumers share information with a platform and the platform sells information about its users. They show that information technologies that improve the platform's ability to gather data may result in an outcome of excessive downstream disclosure. The focus in our analysis is on changes in the abilities of two asymmetric ad networks to gather and monetize data due to regulation and not due to changing technologies (although our framework could be used to examine technology-driven ad monetization changes as well). However, our aim is to identify the differential changes in profits for market stakeholders, rather than to determine the extent of disclosure. In that sense, our analysis is complementary to Kirpalani and Philippon (2020).

Our framework can also be flexibly adopted to derive additional implications. As examples, we demonstrate that (i) a relative increase in the collective market size of large publishers (for instance, due to a number of smaller publishers banding together as an alliance) may have detrimental effects on (other) small publishers, and (ii) a regulation that has asymmetric effects on ad networks can benefit some publishers while harming others.

The remainder of the paper proceeds as follows. Section 2 provides a brief summary of the online ad industry. Section 3 presents our base model comprising, initially, of publishers and two asymmetric ad networks. Publishers differ along two dimensions: size and their preferences for contracting with the two networks. The networks differ in terms of the revenues they can generate for publishers and advertisers. The equilibrium of the base model is derived in Section 4. Section 5 considers the case where large publishers can choose between the automated and negotiated markets and a benchmark case examining the effects of new data restrictions is solved. Section 6 extends the solution to cases where the regulatory change has differential effects on ad networks, and Section 7 examines the impact of a change in the relative sizes of the small and large publisher markets. Section 8 incorporates advertisers into the framework, and Section 9 presents two extensions. Section 10 concludes. Proofs are relegated to the appendix.

2 Ad Industry

Digital publishers can gain ad revenues by interweaving "ad space" into their content (whether in the form of actual spaces on website or apps, or in other forms, such as popups and links), and then selling this ad space to advertisers, either directly or through ad networks. Advertisers, in turn, seek to gain ad "impressions" – views and clicks by users – from certain user demographics. Ad networks – intermediaries that, by signing on multiple publishers and aggregating their ad spaces, specialize in matching advertisers with the user demographics they seek—emerged as multi-sided platforms (linking publishers, advertisers, as well as other ad networks through ad exchanges) that would more efficiently monetize publishers' ad spaces by auctioning those spaces to a large number of potential advertisers (Zawadzinski, 2018). Examples of ad networks include the Google Display Network, YouTube Advertising, Facebook Ads, Yahoo Gemini, AdCash, One by AOL, among others.

There are different types of ad networks based on the publishers they serve and the pricing mechanisms they utilize. For example, so-called premium ad networks seek to contract with more popular publishers, possibly on an individualized basis; certain vertical ad networks tend to focus on topic-specific publishers (e.g. technology, business, fashion, etc); other specialized ad networks serve only certain types of advertisements (e.g., video ads, interactive ads, banner ads), or use specific revenue-sharing mechanisms on the publisher side, and/or ad-billing triggers (cost-per-click, cost-per-impression, among others) on the advertiser side (Zawadzinski, 2018).

As part of the matching process between a publisher and advertisers, and the auction for a publisher's ad space, the publisher may have the ability to control several aspects of the process, such as the minimum price of their ad space, the brands that can or cannot advertise, the type of ads that can be used, among others. When an ad is selected and subsequently presented to a user, the publisher earns a portion of the ad revenue that may be generated, as a function of the specified requirements for and level of user interaction. For example, publishers may retain 80% of the ad revenue generated through Google's Ad Manager, not accounting for ad network commissions on the ad-buying side (Stewart, 2020), 68% of the revenue generated when using Google AdSense, accounting for both ad-selling and ad-buying commissions,[10] and 55% from YouTube Advertising (Spangler, 2013).

To be more specific, in the display advertising marketplace, for instance, there are numerous providers of digital ad services for publishers and advertisers, some specializing in ad space buying, some in ad space selling, and some in both. For example, Google Ads is an ad-buying platform, where advertisers can buy Google search ads, YouTube ads, as well as display ads that appear on non-Google websites and apps. Google Ad Manager is an ad-selling platform, where publishers can choose to sell their ad spaces. Ad selling and buying markets have nuances. Advertisers may pay an ad-buying platform only when a user interacts with an ad (e.g., by clicking the ad, filling out a form or completing a purchase). Some ad-selling platforms may elect to pay publishers for the ad spaces they sell, regardless of subsequent user interactions with ads on their spaces, by utilizing past performance information and user traffic assessments to estimate the revenue a publisher's ad space may generate (the difference resembles real-estate agents earning a commission from matching a landlord's property with a tenant versus outright renting the landlord's property themselves and then subletting it).

In 2019, Google Ads, an ad-buying network, reportedly retained 14% of advertiser spend; Google Ad Manager, an ad-selling network, reportedly collected a commission of 20%. As

[10] See, e.g., `https://adsense.googleblog.com/2010/05/adsense-revenue-share.html`.

a result, by selling ad space and matching with advertisers through the broader Google ad network, publishers retain around 68% of the revenue their ad spaces generate, i.e., of the total amount that advertisers paid for their ad spaces through Google Ads (Hsiao, 2020). The facts that there are separate sell-side and buy-side networks, that one publisher may end up selling ad space to a number of advertisers, and that, conversely, one advertiser may end up buying ad space from a number of publishers, motivate our initial model's setup of considering those two sides of the ad market, the buying and selling sides, separately—focusing on how the flow of ad dollars to or from either of those sides could differentially affect the profits of market stakeholders.

To match ad spaces with users, ad networks utilize user data, both to construct user profiles for purposes of matching users with ads, and to assess user interaction with ads and whether those interactions converted into sales. For instance, user profiles can enable ad networks to fine-tune the targeting of ads, offering more granular user segments for advertisers to choose; similarly, through the use of cookies, web beacons, and tracking pixels, ad networks can track users' browsing behaviors, to assess whether or not they completed the check-out process for purchasing a product after interacting with an ad. These techniques can improve the matching between advertisers and the user demographics they seek, both by improving the accuracy and granularity of user segments and by enabling the network to evaluate which ads are most likely to generate desirable interactions, such as a purchase, from a particular set of users. The latter, in turn, enables the ad network to both present users with more relevant ads, and to optimize the revenue generated from a publisher's ad space, which may subsequently increase the publisher's revenue.

A stricter ruleset governing data utilization by ad networks may limit their ability to match advertisers with users, and thus their ability to monetize ad spaces, and, consequently, the revenues reaped by publishers may decline. Such a stricter ruleset may come from policymakers (e.g., the EU's GDPR[11]) but also from private-sector platforms. An example of the latter is Intelligent Tracking Prevention (ITP)—an Apple, Inc. led initiative to block third-party trackers

[11]In addition to GDPR, there has also been recent activity in the space in the United States. For instance, on December 14, 2020, the Federal Trade Commission (FTC) announced it issued orders under Section 6(b) of the FTC Act to nine social media and video streaming companies (Amazon.com Inc., ByteDance LtD., Discord Inc., Facebook Inc., Reddit Inc., Snap Inc., Twitter Inc., WhatsApp Inc., and Youtube LLC) to learn more about their data practices, particularly for targeting ads, including detail on all material changes made by each company to comply with the GDPR, including whether those changes applied only to EU users.

of user browsing activities. ITP works by collecting statistics on "resource loads as well as user interactions such as taps, clicks, and text entries" and groups those statistics by top privately controlled domains. The system then determines which cookies (of any type) have the ability to track user activity across sites, using a specific time window (e.g., 24 hours) in which those cookies may remain available in third-party contexts (on other, unrelated web properties). "If the user interacted with example.com [in] the last 24 hours, its cookies will be available when example.com is a third-party. This allows for 'Sign in with my X account on Y' login scenarios . . . [Users can] stay logged in even if they only visit a site occasionally while restricting the use of cookies for cross-site tracking" (Wilander, 2017). After the time window, the cookies can still be referenced for login purposes for a longer time window, but no longer for cross-site tracking.

ITP, as well as other recent efforts,[12] including both private-sector initiatives in the app ecosystem, as well as public-sector policies such as GDPR, may advantage some ad networks (particularly, ad networks that can collect user data from other sources or that interact with users in other contexts). This is further evidenced in recent court filings (see, e.g., Davis, 2020) and in academic research (Johnson et al., 2020).

Against this industry backdrop, our model assumes that ad networks can be affected by a stricter data ruleset in terms of the ad revenues they can generate from publishers' ad spaces, and, importantly, that such a stricter ruleset may differentially affect ad networks' monetization efforts. In addition, although, publishers and advertisers may choose to sell and buy ad spaces on more than one ad network (Hsiao, 2020), if returns from one network exceed those of others, or if using more than one network does not result in significant marginal gains, they may choose to work with a single network.[13] In our model, both for simplicity and because our focus is on the downstream effects from a stricter data ruleset, we assume that publishers and advertisers work with a single network. Finally, our model accounts for the common industry practice that different advertisers and publishers may be treated differently by ad networks, as a function of their sizes, industry alliances, and popularity.[14] Specifically, we assume that larger publishers

[12]Apple's iOS 14 operating system, for instance, proposes to use pop-up windows to users to opt out of ad tracking by third-party services. In its 2020 4th quarter earnings call, Facebook CFO Dave Wehner stated: "We also expect to face more significant ad targeting headwinds in 2021. This includes the impact of platform changes, notably iOS 14, as well as the evolving regulatory landscape" (`https://www.engadget.com/facebook-q4-2020-earnings-215456054.html`).

[13]See, e.g., `https://wiki.awin.com/index.php/Your_Affiliate_Programme:_Single_or_Dual_Network`.

[14]Examples include specifically-negotiated deals for popular video game streamers who attract significant user traffic (e.g., `https://www.theverge.com/2020/6/22/21298963/`

and advertisers (in terms of traffic and ad spend, respectively) may negotiate individualized terms with ad networks, whereas smaller publishers and advertisers are relegated to using the networks' automated systems for ad space selling and buying.

3 Base Model

Consider a market consisting of two asymmetric ad Networks (N) and a continuum of Publishers (P). The two ad networks differ in terms of the amount of revenue they can generate for publishers. Each network's capability to generate ad revenue is further influenced by the regulatory environment concerning data usage. As a starting point, we consider the impact of an increase in data protection that hampers the ability of networks to generate ad revenues to be negative and to proportionally affect the ad networks' generated revenues. Accordingly, we denote a θ-network as one which generates a revenue of $(1-d)R_\theta$ from a partnership with a given publisher, where $d \in [0,1)$, $\theta \in \{H(igh), L(ow)\}$, and $R_H > R_L > 0$. The parameter d captures the impact of the regulatory environment concerning data usage on networks' revenues R_θ, such that a rise in d, triggered by new data restrictions, entails a proportional decrease in ad revenues.[15] When $d = 0$, a data regulation has no impact on ad network's revenue.

Publishers are uniformly distributed along the unit interval, identified by their location $x \in [0,1]$ that represents a publisher's preference towards one of the ad networks. Such preferences may arise due to technological compatibility and existing partnerships or familiarity. For succinctness, we refer to a publisher with a preference parameter x as *publisher* x. Each of the two ad networks are located at the opposite ends of the unit interval, with network L at $x = 0 \equiv x_L$ and network H at $x = 1 \equiv x_H$. Publishers incur a 'preference cost' $t > 0$ over the unit distance. Thus, when contracting with network θ, publisher x incurs a cost of $t|x_\theta - x|$.

Ad networks can incentivize publishers to partner with them by making take-it-or-leave-it offers. Contracting with publishers takes the form of revenue-sharing agreements, with publishers

`ninja-shroud-mixer-facebook-gaming-twitch`) and publisher or advertiser consortia for negotiating higher revenue-sharing terms, including the 2019 proposed Journalism Competition and Preservation Act.

[15] The parameter d need not rise continuously or linearly in changes in data protection (or even rise at all). Our underlying premise is that it rises for *some* changes, and our analysis concerns those types of changes. Such changes may include government and private-sector restrictions on the types of data that can be used for marketing purposes (e.g., for matching ads with users). See, for instance, `https://www.adweek.com/programmatic/criteos-revenues-dip-5-as-it-counts-the-cost-of-apples-cookie-blocking-technology-and-gdpr/`.

receiving a share of the ad revenue generated on their content. Among other things, publishers' shares depend on their chosen network and the type of ad market in which they participate. We consider two types of markets, denoted by $\mu \in \{a(utomated), n(egotiated)\}$. In automated markets, the revenue-sharing terms offered by the networks are uniform to all participants. On the other hand, in the negotiated market, networks are able to offer individually-tailored terms to participants. That is, in the *negotiated* market, revenue-sharing terms may depend on a publisher's preference parameter ('location' on the unit interval) $x \in [0, 1]$, whereas in the *automated* market all publishers are offered the same revenue-sharing terms irrespective of their preference parameters. Let $\alpha_\theta^\mu(x) \in [0, 1]$ denote the ad revenue share offered to a publisher x contracting with network θ in market μ. Accounting for their preferences towards the publishers, the payoff for publisher x is given by:

$$\Pi_P(x; \theta, \mu) = \alpha_\theta^\mu(x)(1 - d)R_\theta - t|x_\theta - x|, \quad (1)$$

Being the residual claimant of the ad revenues, the network receives a payoff of $(1 - \alpha_\theta^\mu(x))\,(1 - d)R_\theta$ from this contract. Let $M(\theta, \mu)$ denote the share of publishers that contract with network θ in market μ. Then the aggregate payoff of network θ in market μ is given by:

$$\Pi_N(\theta, \mu) = \int_{M(\theta,\mu)} (1 - \alpha_\theta^\mu(x))\,(1 - d)R_\theta dx \quad (2)$$

We make the following assumptions:

Assumption 1 *:* $t > R_H - R_L > 0$

Assumption 1 ensures that ad networks are viable. It both places a lower bound on preference costs towards the two ad networks and requires them to be sufficiently high to ensure that at least some publishers are induced to contract with network L.

Assumption 2 *:* $t < \frac{1-d}{3}(R_H + R_L)$

Assumption 2 places an upper bound on preference costs and requires them to be sufficiently low to ensure that all publishers are induced to contract with at least one of the networks. A combination of Assumptions 1 and 2 further yields that $(1 - d)R_L > t$, ensuring that both networks, if operating alone, are able to cover the market.

In the proceeding, we assess the effects of a decline in ad revenues due to a more restrictive data regime. As a benchmark, we begin by considering the automated and negotiated markets separately in the next section.

4 Equilibrium Characterization

4.1 Automated Market

By definition, in their automated markets, networks offer uniform revenue-sharing terms to all participating publishers, that is, $\alpha_\theta^a(x) = \alpha_\theta^a$ for all $x \in [0,1]$. With this in mind, it is straightforward to see from (1) that publisher x's payoffs from contracting with the networks L and H are given by:

$$\Pi_P(x;\theta,a) = \begin{cases} \alpha_L^a(1-d)R_L - tx & \text{if} \quad \theta = L \\ \alpha_H^a(1-d)R_H - t(1-x) & \text{if} \quad \theta = H. \end{cases} \tag{3}$$

Publisher x will choose network L when the payoff from doing so exceeds the payoff from contracting with network H. We note that $\Pi_P(x;L,a)$ is decreasing in x and $\Pi_P(x;H,a)$ is increasing in x. Assuming that preference costs satisfy Assumptions 1 and 2, both networks are viable in equilibrium, whereby there exists $\tilde{x}^a$ such that all publishers with $x < \tilde{x}^a$ contract with network L whereas those with $x \geq \tilde{x}^a$ contract with network H (breaking indifference in favor of H without loss of generality). Hence, network L's market share is $M(L,a) = [0, \tilde{x}^a)$ and H's market share is $M(H,a) = [\tilde{x}^a, 1]$. The cutoff $\tilde{x}^a$ solves the condition $\Pi_P(x;L,a) = \Pi_P(x;H,a)$ which yields:

$$\tilde{x}^a = \frac{1}{2} - \frac{(1-d)(\alpha_H^a R_H - \alpha_L^a R_L)}{2t}. \tag{4}$$

As can be immediately seen from (4), L's market share is smaller than H's market share as long as $\alpha_H^a R_H > \alpha_L^a R_L$, a condition that will be met in equilibrium. Further, $\tilde{x}^a$ is decreasing in α_H^a and increasing in α_L^a, implying that each network gains market share when increasing the revenue-sharing term it offers publishers, α_θ^a, and loses market share whenever the competing network does so. Plugging $M(\theta, a)$ in (2), network L sets α_L^a to maximize

$$\Pi_N(L, a) = \int_0^{\tilde{x}^a} (1 - \alpha_L^a)(1 - d)R_L dx, \tag{5}$$

and network H sets α_H^a to maximize

$$\Pi_N(H, a) = \int_{\tilde{x}^a}^1 (1 - \alpha_H^a)(1 - d)R_H dx. \tag{6}$$

where $\tilde{x}^a$ is given by (4) and varies with both α_H^a and α_L^a. Differentiating (5) and (6) with respect to α_L^a and α_H^a, respectively, and solving for the networks' profit-maximizing revenue-sharing terms in the automated market yields:

$$\alpha_L^{a,*} = \frac{2}{3} + \frac{(1 - d)R_H - 3t}{3(1 - d)R_L}, \text{and} \tag{7}$$

$$\alpha_H^{a,*} = \frac{2}{3} + \frac{(1 - d)R_L - 3t}{3(1 - d)R_H}. \tag{8}$$

Given that $R_H > R_L$, a straightforward comparison between (7) and (8) reveals that $\alpha_H^{a,*} < \alpha_L^{a,*}$, implying that network L offers better revenue-sharing terms to its publishers than its competitors. However, because of the lower ad revenues L generates from publishers' ad spaces, these more favorable terms do not necessarily translate to higher payoffs for the publishers. In fact, it can readily be verified that $\alpha_H^{a,*}(1 - d)R_H > \alpha_L^{a,*}(1 - d)R_L$. That is, if publishers did not have heterogeneous preferences for the two networks, they would choose to contract with network H. By plugging (7) and (8) in (3), in equilibrium, the payoff of publisher x depends on the market μ and the preference parameter x, and is given by:

$$\Pi_P^*(x; a) = \begin{cases} \frac{(1-d)(R_H+2R_L)}{3} - t(1 + x) & \text{if} \quad 0 \le x < \tilde{x}^{a,*} \\ \frac{(1-d)(2R_H+R_L)}{3} - t(2 - x) & \text{if} \quad \tilde{x}^{a,*} \le x \le 1, \end{cases} \tag{9}$$

where

$$\tilde{x}^{a,*} = \frac{1}{2} - \frac{(1 - d)(R_H - R_L)}{6t}. \tag{10}$$

The expression for $\tilde{x}^{a,*}$ in (10) reveals that, despite offering more favorable revenue-sharing terms, network L contracts with a smaller proportion of publishers. It can also be surmised from (10) that both networks are viable in equilibrium whenever $t > \frac{1-d}{3}(R_H - R_L)$, a condition that is always met whenever Assumption 1 is satisfied. Finally, $\alpha_\theta^{a,*}$ and $\tilde{x}^{a,*}$ derived above can be substituted in (2) to obtain the networks' equilibrium profits:

$$\Pi_N^*(\theta, a) = \begin{cases} \frac{(3t-(1-d)(R_H-R_L))^2}{18t} & \text{if} \quad \theta = L, \\ \frac{(3t+(1-d)(R_H-R_L))^2}{18t} & \text{if} \quad \theta = H. \end{cases} \tag{11}$$

Since $\alpha_L^{a,*} > \alpha_H^{a,*}$ and $\tilde{x}^{a,*} < \frac{1}{2}$, unsurprisingly, network L's payoff is lower than H's. Given the preceding profit characterization, we can now assess the effects of a stricter data regime, as represented by the parameter d, in the automated market.

Proposition 1 *New data restrictions have the following effects in the automated markets: (i) Publishers' advertising revenue shares and profits decrease. (ii) The proportion of publishers that contract with the L (H) network as well as the L (H) network's profit increase (decrease).*

The first part of Proposition 1 highlights the adverse effects of the stricter data regime on publishers, pointing to profit declines as a consequence of networks' diminished abilities to generate ad revenues. Intuitively, since the regulatory tightening reduces networks' ad revenues, they attempt to recoup some of their losses by offering worse revenue-sharing terms to publishers. The second part of Proposition 1 notes that, somewhat surprisingly, a stricter data ruleset has an asymmetric effect on the two networks. Specifically, the stricter data regime enables the L network to contract with a larger proportion of publishers and earn higher profits (in aggregate) than before, although its larger competitor, along with all of the publishers, incur profit losses. To gain some intuition for this result, note that from the expression of $\tilde{x}^{a,*}$ in (10), it is straightforward to see that it is increasing in d. This is because a proportional decrease in ad revenues due to a tightening in the regulatory environment improves the relative positioning of the L network and intensifies competition over publishers. As a consequence, in the automated market, the L network is a net beneficiary of the stricter data regime, albeit in part at the expense of publishers that contract with it.

4.2 Negotiated Market

In the negotiated market, networks offer individualized revenue-sharing terms to publishers, based on their preference parameter x. The payoffs to publisher x from contracting with each of the networks are given by:

$$\Pi_P(x;\theta,n) = \begin{cases} \alpha_L^n(x)(1-d)R_L - tx & \text{if} \quad \theta = L \\ \alpha_H^n(x)(1-d)R_H - t(1-x) & \text{if} \quad \theta = H. \end{cases} \tag{12}$$

Similarly to the automated markets, publisher x will choose the network that will result in a higher net payoff. However, unlike the automated market structure, competition over individual publishers results in Bertrand-like dynamics. That is, the equilibrium outcome is one in which one of the networks is driven to offer the publisher all of the ad revenues generated, with the other network offering slightly better terms in net, accounting for the publisher's preference parameter. Given continuity of the revenue-sharing terms, we break publisher indifference towards the networks in favor of the network that can offer better terms in net, accounting for the publisher's preference parameter. Competition is thus most intensified when networks offer their best terms and retain none of the ad revenues, that is, when $\alpha_H^n(x) = \alpha_L^n(x) = 1$. At these terms, the marginal publisher that receives the same net payoffs from contracting with either of the two networks is given by:

$$\tilde{x}^{n,*} = \frac{1}{2} - \frac{(1-d)(R_H - R_L)}{2t}. \tag{13}$$

Applying Assumption 1, the expression in (13) implies that both networks are viable in the negotiated market, that is, $\tilde{x}^{n,*} \in (0,1)$. It is also notable that $\tilde{x}^{n,*} < \tilde{x}^{a,*}$, i.e., the proportion of publishers contracting network L is smaller in the negotiated than in the automated market.

For publishers with preference parameters $x \neq \tilde{x}^{n,*}$, one of the networks offers a revenue share that is strictly less than 1. To see this, first consider publishers with preferences $x < \tilde{x}^{n,*}$. Since $\Pi_P(x;L,n)$ ($\Pi_P(x;H,n)$) is decreasing (increasing) in x, if $\alpha_H^n(x) = \alpha_L^n(x) = 1$ is offered to these publishers, they would be strictly better off contracting with network L, whereby L could decrease its revenue-sharing term. Accordingly, the optimal revenue-sharing terms network L offers are given by:

$$\alpha_L^{n,*}(x) = \begin{cases} \frac{(1-d)R_H - t(1-2x)}{(1-d)R_L} & \text{if} \quad 0 \leq x < \tilde{x}^{n,*}, \\ 1 & \text{if} \quad \tilde{x}^{n,*} \leq x \leq 1. \end{cases} \tag{14}$$

Analogously, network H offers the following terms:

$$\alpha_H^{n,*}(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x < \tilde{x}^{n,*}, \\ \frac{(1-d)R_L + t(1-2x)}{(1-d)R_H} & \text{if} \quad \tilde{x}^{n,*} \leq x \leq 1. \end{cases} \tag{15}$$

It follows from (14) and (15) that $\alpha_L^{n,*}(x)$ is increasing in x and $\alpha_H^{n,*}(x)$ is decreasing in x; that is, the marginal publisher $\tilde{x}^{n,*}$ receives the most favorable terms, whereas publishers with preference parameters closest to 0 (L) or 1 (H) receive the least favorable offers. Publishers' profits in the negotiated market are given by:

$$\Pi_P^*(x; n) = \begin{cases} (1-d)R_H - t(1-x) & \text{if} \quad 0 \leq x < \tilde{x}^{n,*}, \\ (1-d)R_L - tx & \text{if} \quad \tilde{x}^{n,*} \leq x \leq 1. \end{cases} \tag{16}$$

The marginal publisher with $x = \tilde{x}^{n,*}$ obtains the highest profit in this market. Substituting the expressions for $\alpha_\theta^{n,*}$ and $\tilde{x}^{n,*}$ in (2), the networks' profits in the negotiated market are:

$$\Pi_N^*(\theta, n) = \begin{cases} \frac{(t-(1-d)(R_H - R_L))^2}{4t} & \text{if} \quad \theta = L, \text{ and} \\ \frac{(t+(1-d)(R_H - R_L))^2}{4t} & \text{if} \quad \theta = H. \end{cases} \tag{17}$$

Given the above profit characterizations, we can now assess the effects of a change in the restrictiveness of the data regime that affects the networks' abilities to generate ad revenues, as represented by a rise in the parameter d, in the negotiated market.

Proposition 2 *New data restrictions have the following effects in the negotiated market: (i) The revenue shares offered to publishers with* $x \in [0, \tilde{x}^{n,*}) \cup (\frac{1}{2}, 1]$ *decrease, while those offered to publishers with* $x \in [\tilde{x}^{n,*}, \frac{1}{2}]$ *increase. (ii) Publishers' profits decrease. (iii) The proportion of publishers that contract with network L (H) and network L's (H's) profit increase (decrease).*

Proposition 2 highlights the asymmetric impact of a stricter data ruleset on the advertising revenue-sharing terms that the networks offer. As ad revenue streams diminish following a rise in d, competition over publishers with preference parameters $x \in [\tilde{x}^{n,*}, \frac{1}{2}]$ intensifies. Intuitively, the proportional reduction in ad revenues entails a larger magnitude decrease to the advertising revenues generated by the H network—this means the H network has to offer better terms in order to attempt to retain the publishers in $x \in [\tilde{x}^{n,*}, \frac{1}{2}]$, with preference parameters closest to the marginal publisher that is indifferent between the two networks. Despite these dynamics, with the overall revenue pie shrinking due to the rise in d, all publishers' profits decrease. Overall, similarly to the automated markets, a stricter data ruleset benefits the L network while publishers and the H network incur losses.

4.3 Comparing the Two Market Outcomes

We next compare the outcomes for the stakeholders in the automated and negotiated markets. For purposes of an apples-to-apples comparison of profits, we consider the payoffs of publishers from selling a unit of ad space in each of the markets. We then consider alternate settings, include where large publishers can choose between the two markets, in the proceeding sections. We have the following result.

Proposition 3 *Comparing the equilibrium outcomes under the negotiated market to the automated markets: (i) The share of publishers that contract with the L network is smaller. (ii) The revenue shares and profits of publishers with preference parameters $x \in [0, \tilde{x}_{IC}^{n,*}]$ are higher while those with $x \in (\tilde{x}_{IC}^{n,*}, 1]$ are lower, where $\tilde{x}_{IC}^{n,*} = 1 - \frac{(1-d)(R_H - R_L)}{3t}$. (iii) Network L's profit is lower. (iv) Network H's profit is lower for sufficiently high values of t and higher otherwise.*[16]

Proposition 3 highlights how the specific market mechanism for selling ad space (automated vs negotiated) leads to different outcomes for market participants. For instance, as expected, more aggressive competition over publishers in the negotiated market enables the H network to capture a larger market share. Figure 1 illustrates how each publisher's profit varies in the two markets according to their preference parameter x. All publishers, except those with $x > \tilde{x}_{IC}^{n,*}$, derive higher profits in the negotiated market than in the automated market. This is because the more intense competition between the two networks in the negotiated market leads to

[16]Specifically, Network H's profit is lower for $t > 1.27(1-d)(R_H - R_L)$ and higher otherwise. Note that given Assumption 1, depending on the value of d, this inequality may always be satisfied; however, our focus here is not on specific parameterizations of d but rather on the effects of a *change* in d.

better terms for most publishers. The exceptions are those publishers exhibiting the strongest preference for (located closest to) the H network, who would benefit from pooling with other publishers (located closer to the middle in terms of their preference for the ad networks) in H's automated market, due to its uniform terms.

Proposition 3 also highlights the different outcomes for the networks. Since the L network has a smaller market share in the negotiated market and is forced to offer higher revenue shares to publishers, unsurprisingly its profit is always lower than in its automated market. For the H network, the impact depends on the model's parameterization; specifically, on the preference cost or the degree of the networks' horizontal differentiation, as represented by t. For sufficiently high values of t, the H network's profit is also lower in the negotiated market than in its automated counterpart. However, if the networks lack horizontal differentiation, it is possible for the H network's profit to be higher in the negotiated market. This is due to two opposing tradeoffs: On the positive side, H is able to gain market share from L when revenue-sharing terms are individualized. On the negative side, individualized revenue-sharing terms cut into H's profit. In the absence of horizontal differentiation, the H network is able to effectively push the L network out of the negotiated market, due to its vertically superior service (although the existence of the L network still constrains the H network's revenue-sharing terms), with H's profit being higher in net in comparison to its automated market.

5 A Model with Two Markets

In this section, we extend our base model to consider a framework where the automated and negotiated markets operate in parallel. We distinguish between small and large publishers based on how their revenue-sharing terms are determined. Specifically, small publishers participate in automated markets operated by each ad network, where revenue-sharing agreements are uniform for participants. Large publishers, on the other hand, may receive better, individually tailored terms. Figure 2 depicts the relationship between publishers and ad networks. In this setup, the payoffs of the small publishers are the same as in Section 4.1.

The payoffs of large publishers, on the other hand, are slightly different from those characterized in Section 4.2. Given ad networks' offers in each market, large publishers decide

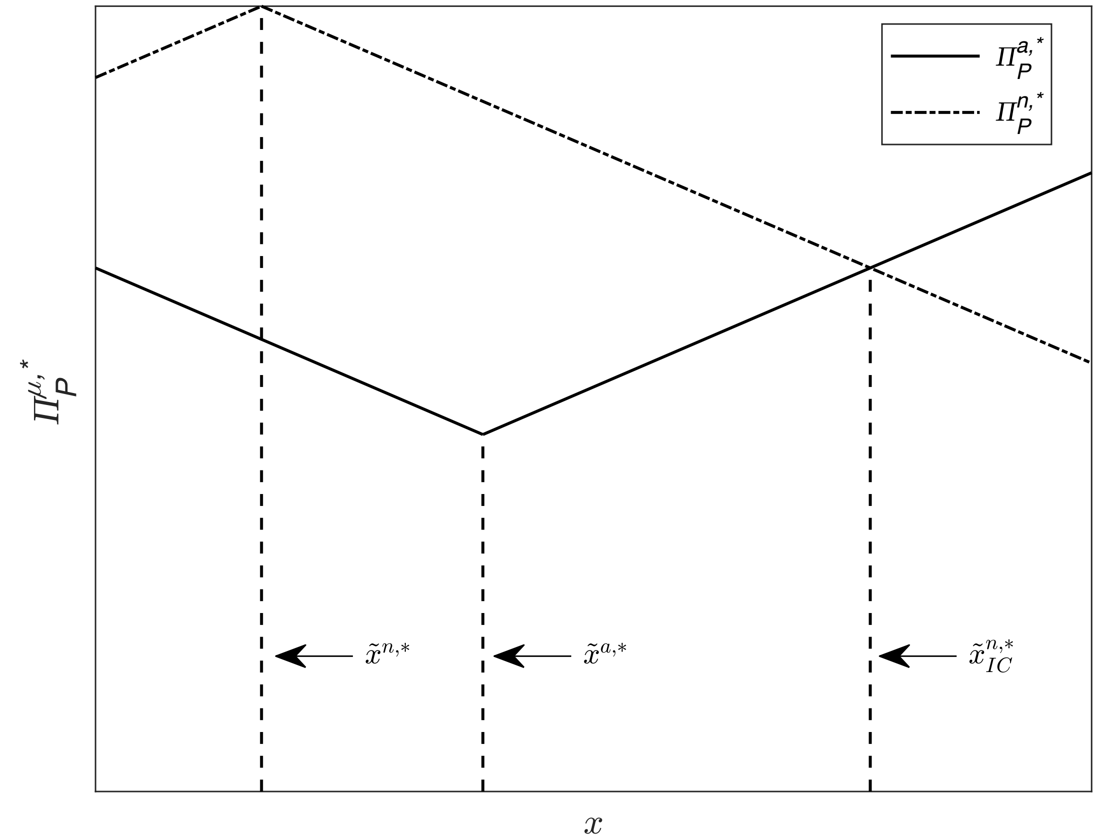

Figure 1: Publishers' equilibrium profits

on a market in which to participate. As a result, large publishers may decide to switch over to the automated markets if doing so can provide them with better terms. As established in Proposition 3, large publishers with preference parameters $x > \tilde{x}_{IC}^{n,*}$ can indeed receive better revenue-sharing terms by switching to H's automated market. In equilibrium, network H must account for this participation constraint when determining its profit-maximizing terms in both its automated market and in the negotiated market. For this subset of publishers, the participation constraint is binding, such that network H sets $\alpha_H^n(x) = \alpha_H^{a,*}$, that is, the network matches the terms those publishers can receive in the automated market. Consequently, while the equilibrium revenue-sharing terms of the L network in the negotiated market are still given

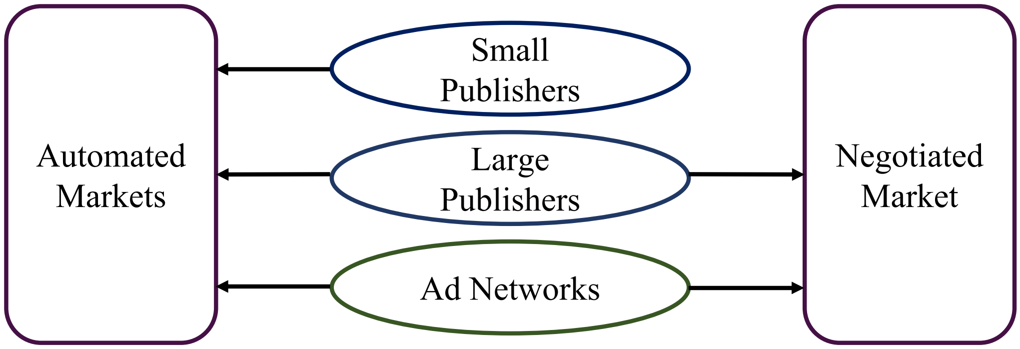

Figure 2: A depiction of the relationship between publishers and ad networks

by (14), those of the H network become:

$$\alpha_H^{n,*}(x) = \begin{cases} 1 & \text{if} \quad 0 \le x < \tilde{x}^{n,*}, \\ \frac{(1-d)R_L + t(1-2x)}{(1-d)R_H} & \text{if} \quad \tilde{x}^{n,*} \le x \le \tilde{x}_{IC}^{n,*}, \\ \frac{2}{3} + \frac{(1-d)R_L - 3t}{3(1-d)R_H} & \text{if} \quad \tilde{x}_{IC}^{n,*} \le x \le 1. \end{cases} \tag{18}$$

For simplicity, we assume at this stage that the market for large publishers is significantly smaller than the market for small publishers (with Lebesgue measure zero). Consequently, the fact that some large publishers have a binding participation constraint does not affect the revenue-sharing terms in the automated markets. We then relax this assumption in Section 7. Therefore, given the revenue-sharing terms in (14) and (18), large publishers' equilibrium profits are given by

$$\Pi_P^*(x;n) = \begin{cases} (1-d)R_H - t(1-x) & \text{if} \quad 0 \le x < \tilde{x}^{n,*}, \\ (1-d)R_L - tx & \text{if} \quad \tilde{x}^{n,*} \le x \le \tilde{x}_{IC}^{n,*}, \\ \frac{(1-d)(2R_H + R_L)}{3} - (2-x)t & \text{if} \quad \tilde{x}_{IC}^{n,*} \le x \le 1, \end{cases} \tag{19}$$

with the following profits for the two ad networks from large publishers:

$$\Pi_N^*(\theta, n) = \begin{cases} \frac{(t-(1-d)(R_H - R_L))^2}{4t} & \text{if} \quad \theta = L, \\ \frac{((1-d)(R_H - R_L) + 3t)(5(1-d)(R_H - R_L) + 3t)}{36t} & \text{if} \quad \theta = H. \end{cases} \tag{20}$$

We are now ready to assess the effects of reductions in ad revenues due to a change in the regulatory environment for data usage, as represented by the parameter d, in this extended framework.

Corollary 1 *New data restrictions have the following effects: (i) The share of publishers that contract with the L (H) network as well as the L (H) network's profit increase (decrease). (ii) Revenue-sharing terms worsen for all publishers except large publishers with $x \in (\tilde{x}^{n,*}, \frac{1}{2})$. (iii) In net, each publisher's profit decreases.*

The results in Corollary 1 are direct implications of Propositions 1 and 2. Table 1 provides a summary of Corollary 1.

Network	*H*	*L*
Market Share	Decreases	Increases
Profit	Decreases	Increases
Publishers	*Small*	*Large*
Revenue-Sharing Terms	Decreases	Mixed
Profit	Decreases	Decreases

Table 1: Summary of the effects of reductions in ad revenues on networks and publishers

We next assess the relative effects on publishers' profits by defining and then determining a unitless elasticity measure. This measure captures the percentage decline in profit from selling a unit of ad space that results from a percentage (proportional) decline in ad revenues due to new data restrictions. Doing so makes it possible to identify where, ceteris paribus, the brunt of the negative effects among publishers are incurred.

Definition 1 *We denote the percentage change in a market stakeholder's profit π that results from a percentage reduction in ad revenues due to new data restrictions, as captured by d, by*

$$\epsilon_{\pi,d} = \frac{\%\Delta\pi}{\%\Delta d} = \frac{\partial\pi}{\partial d} \cdot \frac{d}{\pi}. \tag{21}$$

That is, $\epsilon_{\pi,d}$ is the profit elasticity with respect to ad-revenue degradation.

We compare the elasticity measure in Definition 1 along two dimensions: (i) across markets, i.e., between small and large publishers participating in the automated and negotiated markets, respectively, and (ii) within the same market type (automated, negotiated) according to publishers' preferences towards the ad networks in that type of market—publishers' preferences, in equilibrium, also determine the network with which they contract.

Definition 2 *A small publisher located at* $x \in [0, 1]$ *is said to be* **similarly** *positioned to a large publisher located at* x*. A publisher, small or large, located at* x *who contracts with* L *is said to be* **analogously** *positioned to a publisher of the same size contracting with* H *located at* $1 - x$*.*

Definition 2 provides a structure for meaningfully comparing profit elasticities along the above two dimensions—across markets and within markets.

Proposition 4 *Ad revenue reductions due to a stricter data ruleset have the following effects:*

1. *The percentage profit declines that small publishers incur are greater than or equal to those incurred by similarly positioned large publishers, and strictly so for* $x < \tilde{x}_{IC}^{n,*}$*.*
2. *Small publishers who contract with network* L *are harmed more than analogously positioned small publishers who contract with* H*; the reverse holds for large publishers.*

Intuitively, ad networks compete over small publishers less aggressively than they do over large publishers. Consequently, a tightening in the regulatory environment for data usage leads to small publishers absorbing more of the subsequent declines in the ad revenues that the networks can generate. Moreover, due to a smaller overall ad revenue pie for the lower-revenue network L, small publishers who contract with it tend to incur larger percentage profit decreases after new data restrictions are imposed. The reverse tends to hold for large publishers because they already extract a larger share of ad revenues; that is, large publishers who contract with the H network may have greater exposure to ad revenue declines than those who contract with the L network. Table 2 summarizes the findings of Proposition 4.

Market Segment	*Small Publishers*	*Large Publishers*
Contract with L	More Harm	Less Harm
Contract with H with $x < \tilde{x}_{IC}^{n,*}$	More Harm	Less Harm
Contract with H with $x > \tilde{x}_{IC}^{n,*}$	Same Harm	Same Harm
Contract With Network	L	H
Small Publishers	More Harm	Less Harm
Large Publishers	Less Harm	More Harm

Table 2: Relative impact of new data restrictions on publishers' profits

It follows from Proposition 4 that the profit declines incurred by small publishers are strictly greater in percentage terms than those incurred by similarly positioned large publishers. An exception to this are those large publishers for which the participation constraint binds, where the percentage profit declines are equal to similarly positioned small publishers.[17] Moreover, small publishers who contract with network L tend to incur higher percentage profit declines than those who contract with H. These findings indicate that small publishers who contract with the 'weaker' ad network tend to take the brunt of the negative effects from a tightening in the regulatory environment for data usage. That is, smaller market actors are particularly susceptible to incurring profit losses from new data restrictions.

Our analysis thus far assumed that new data restrictions trigger a proportional reduction in networks' ad revenues. The next section extends these findings to cases where, consistent with empirical evidence (e.g., Johnson and Shriver, 2019), network H's revenues may be impacted differently than network L's once new restrictions are imposed.

6 Asymmetric Regulatory Impact

We extend the analysis from the base model to cases where the ad revenues that network H is able to generate following a tightening in the regulatory environment are impacted differently from those of network L. Specifically, while the ad revenues that network L can generate decline by a factor d, those generated by network H change by a factor ψd. The ad revenues the L and H networks can generate per publisher subsequently change to to $(1-d)R_L$ and $(1-\psi d)R_H$, respectively. We focus on the following cases: (i) If $\psi \in (0,1)$, then L's revenues decrease more than H's; (ii) if $\psi > 1$, then L's revenues decrease less than H's; and (iii) if $\psi < 0$, then H's revenues increase while L's decline.[18] We make the following assumption:

Assumption 3 *: (i)* $-\frac{R_L}{2R_H} < \psi < \frac{R_H-(1-d)R_L}{dR_H}$, *(ii)* $R_H > \frac{4}{3}R_L$

The upper bound in Assumption 3(i), which is greater than 1, ensures that the ordering of the two networks in terms of revenues does not switch following the imposition of the new data restrictions, such that $(1-\psi d)R_H > (1-d)R_L$ holds. The lower bound in 3(i) along with 3(ii) provide sufficient conditions to ensure that the asymmetric variants (incorporating

[17] Section 9 shows that this equality is driven by the assumption that units of ad space generate equal ad revenues for small and large publishers; once this assumption is relaxed, all small publishers are harmed more than similarly positioned large publishers.

[18] There are additional cases, such as (iv) if $\psi = 0$ then only L's revenues decline; (v) if $\psi \to \infty$, then only H's revenues decline; and (vi) if $\psi = 1$ then H and L's revenues decline proportionally as in the base model.

ψ) of Assumptions 1 and 2 continue to hold as well; that is, that the networks are both viable and the market is covered. The proceeding result examines the interaction between regulatory tightening, d, and the asymmetry in regulatory effects, ψ, in the market for small publishers.

Proposition 5 *New data restrictions with asymmetric changes in the revenues ad networks can generate have the following effects on small publishers in the automated market:*

1. *The share of small publishers that contract with the L (H) network as well as L's (H's) profit decrease (increase) for* $-\frac{R_L}{2R_H} < \psi < \frac{R_L}{R_H}$*, and increase (decrease) otherwise.*
2. *Network L's revenue-sharing term increases if* $\psi < 1 - \frac{3t}{R_H}$ *and decreases otherwise. Network H's revenue-sharing term decreases.*
3. *The profits of all small publishers decline.*

Proposition 5 extends the characterization from the previous section to the case where the change in the regulatory environment differentially impacts the ad networks. Of note, under relatively mild assumptions, each small publisher's profit continues to decrease. This reduction holds even if the H network's ad revenue rises rather than declines after the regulatory change. This is a result of publishers' outside options to H being diminished, which in turn leads to less favorable revenue-sharing terms.

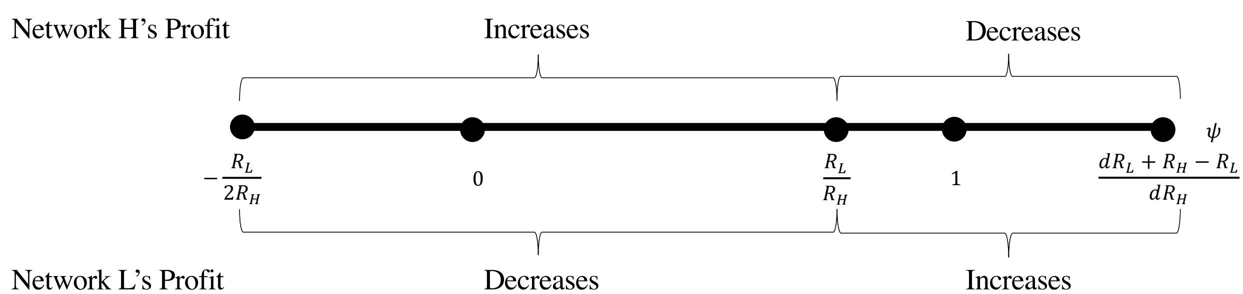

Figure 3: Changes in network profits from publishers vis-à-vis regulatory asymmetry, ψ

Figure 3 summarizes the effects on ad networks' profits from small publishers. The next result extends the analysis to the market for large publishers.

Proposition 6 *New data restrictions with asymmetric changes in the revenues ad networks can generate have the following effects on large publishers in the negotiated market:*

1. *The share of large publishers that contract with the L (H) network as well as L's (H's) profit decrease (increase) for* $\psi < \frac{R_L}{R_H}$, *and increase (decrease) otherwise.*

2. *Network L's revenue-sharing terms increase if* $\psi < 1 - \frac{t(1-2x)}{R_H}$ *and decrease otherwise. Network H's revenue-sharing terms for large publishers with* $x \leq \tilde{x}_{IC}^{n,*}$ *increase if* $\psi > \frac{R_L}{R_L+t(1-2x)}$ *and decrease otherwise; they decrease for large publishers with* $x > \tilde{x}_{IC}^{n,*}$.

3. *The profits of large publishers contracting with network L decrease for* $\psi > 0$ *and increase otherwise. The profits of large publishers contracting with network H decrease.*

Figure 3 therefore also summarizes the effects on ad networks' profits from large publishers. Per Proposition 5, the profits of small publishers decline over the entire range of regulatory asymmetry parameter values ψ that we consider. It is straightforward to see that the combination of parts 3 of Propositions 5 and 6 immediately indicate that given sufficiently small values of ψ, the prior results regarding the relative elasticities of small and large publishers who contract with L continue to hold. That is, in cases where ψ is small or negative (i.e., even if the H network gains after new data restrictions are imposed), those small publishers contracting with L incur larger percentage profit losses than their similarly positioned large counterparts.

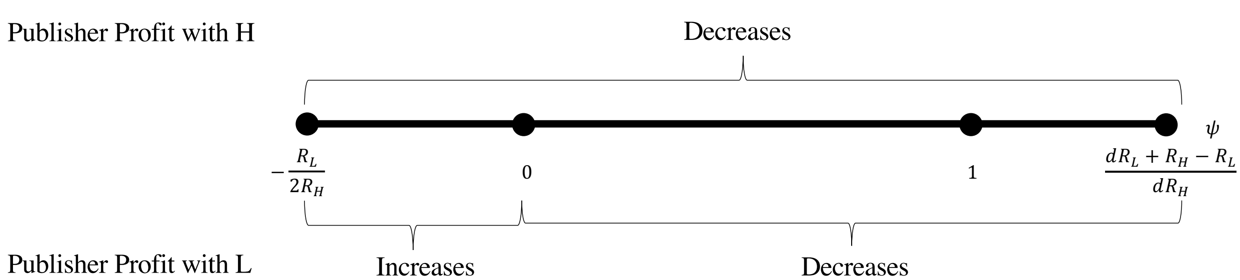

Figure 4: Changes in large publisher profits as a function of regulatory asymmetry, ψ

Figure 4 depicts the effects on the profits of large publishers, as detailed in Proposition 6. In both the markets for small and large publishers, following new data restrictions, if network L incurs a disproportionate reduction in the ad revenues it can generate, then L loses market share. Revenue-sharing terms and publishers' profits tend to decrease all across, except under some degrees of regulatory asymmetry for a range of large publishers who contract with L. Of particular note, the asymmetry associated with the regulatory effects strengthens the market positioning of network H, which then tends to have detrimental effects on the other market participants.

7 Impact of Large Publishers

Our analysis thus far focused on the case where the mass of the large publisher market has Lebesgue measure zero. This assumption meant that the participation constraint for large publishers to take part in the negotiated (and not in the automated) market, while it was binding for some large publishers, did not interact with the H network's decision of how to set its revenue-sharing term in the automated market. While relaxing this assumption does not qualitatively change the results, it provides for interesting comparative statics with respect to the impact on the profits of the different market stakeholders as a result of a change in the relative size of the negotiated market. Such a change may take place when a group of small publishers band together to form an alliance or a consortium and thus switch to become 'large' publishers vis-à-vis the ad networks.

This section considers the case of a positive and changing mass ($\gamma > 0$) for the large publisher market under the base model scenario, that is, where the change in the regulatory environment has a proportional effect on ad revenues ($\psi = 1$). Since the participation constraint for large publishers does not bind for network L, allowing for $\gamma > 0$ does not alter L's maximization problem, that is, it still chooses α_L^a in its automated market to maximize (5). Network H, in contrast, now chooses α_H^a in its automated market to maximize

$$\int_{\tilde{x}^a}^{1} (1 - \alpha_H^a)(1 - d) R_H dx - \gamma \int_{\tilde{x}_{IC}^n}^{1} (\alpha_H^a - \alpha_H^{n,*}(x))(1 - d) R_H dx, \tag{22}$$

where $\tilde{x}^a$ is given in (4), $\alpha_H^{n,*}(x)$ is given in (15) and $\tilde{x}_{IC}^n \geq \tilde{x}^a$ solves $\alpha_H^a = \alpha_H^{n,*}(x)$. As $\alpha_H^{n,*}(x)$ given in (15), $\alpha_H^a - \alpha_H^{n,*}(x)$ is positive for all publishers $x \in [\tilde{x}_{IC}^n, 1]$. Therefore, the second term in (22) captures the opportunity cost network H incurs due to the fact that the participation constraint is binding for some positive mass of large publishers. After accounting for this cost, the optimal revenue-sharing terms and resulting profits for networks and publishers can be derived in an identical manner as Section 6. Equipped with the equilibrium characterization, we can study the impact of increasing the size of large publisher market on market outcomes.

The findings of this analysis are summarized in the following Proposition.

Proposition 7 *Given a rise, ceteris paribus, in the large publisher market size, γ:*

1. *The share of publishers that contract with network L (H) increases (decreases) in the automated market; market shares do not change in the negotiated market.*

2. *Revenue-sharing terms and profits for small publishers decrease; terms and profits do not change in the negotiated market except for decreases for large publishers with $x \in (\tilde{x}_{IC}^{n,*}, 1)$.*

A rise in the size of the market for large publishers therefore entails a worsening in revenue-sharing terms and small publisher profits in the automated market. This follows from the participation constraint for some large publishers that network H faces becoming stricter—as a result, network H shifts more weight to its profit from the negotiated market; consequently, H decreases its revenue-sharing term in the automated market to make it less appealing for large publishers to switch to the automated market. This has a dampening effect on competition for small publishers, resulting in both networks decreasing revenue-sharing terms in their automated markets, allowing network L to expand its share of the small publishers market, but at the same time worsening conditions for small publishers all across.

Proposition 7 has important implications for policies that encourage alliances and consortia among a subset of small publishers (e.g., the proposed Journalism Competition and Preservation Act), in that those publishers that remain outside the consortia will suffer from weaker competition over their ad spaces, and worse revenue-sharing terms and profits as a result.

8 A Model with Advertisers

We have so far omitted explicitly considering the advertiser side of the overarching market, since advertisers' ad campaigns may be distributed to multiple publishers and/or sold through separate, buy-side ad networks. However, our base model setup readily extends to advertisers. Specifically, consider an advertiser that selects an ad network $\theta \in \{L, H\}$ on which to advertise, and pays the network a cost in order to advertise to the publishers to which that network can distribute ads. In exchange for this payment, the advertiser anticipates a return r_θ (e.g., in the form of ad conversions to product purchases). As in the base model, we analogously distinguish between small and large advertisers who participate in automated and negotiated markets, respectively. Advertising costs in the automated market are uniformly determined, where we abstract from auction processes for analytical tractability,[19] whereas they are determined individually for large advertisers in the negotiated market. This distinction implies that larger advertisers are competed over more intensely by the networks. Figure 5 extends Figure 2 to also incorporate small and large advertisers.

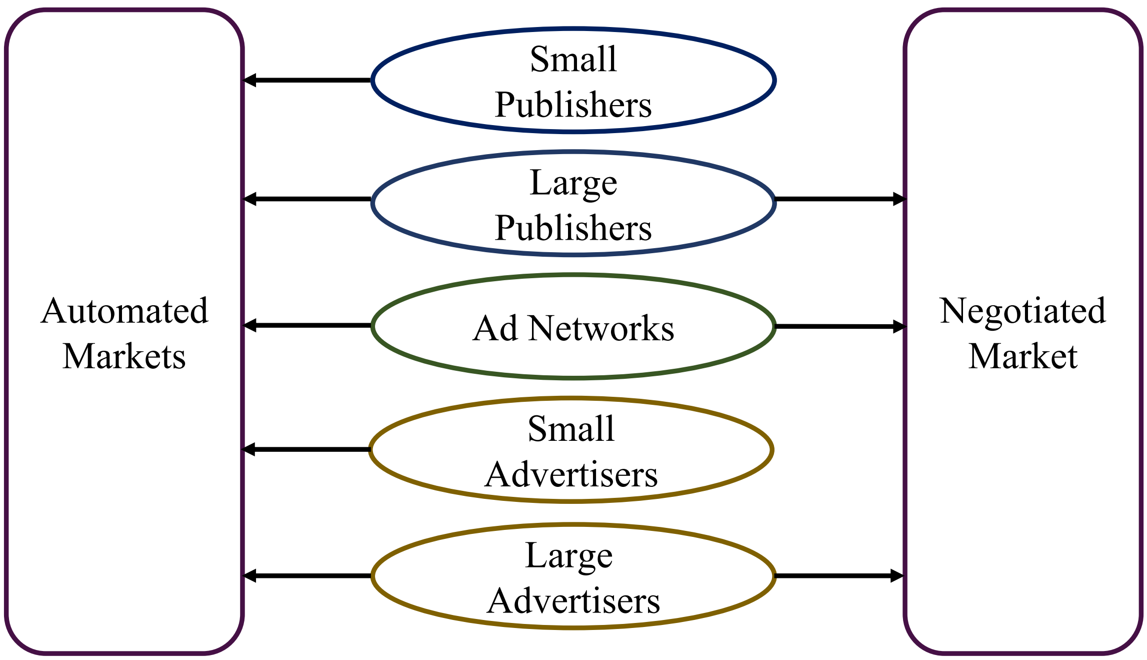

Figure 5: A depiction of the relationship between publishers, advertisers, and ad networks

Let b^{μ}_{θ} denote an advertiser's cost to advertise in market $\mu \in \{a, n\}$. That is, the setup is analogous to the publisher side in the base model, except that in lieu of an ad revenue-sharing term, advertisers incur an advertising cost. Under this setup, an advertiser's profit is given by

$$\Pi_A(\theta, \mu) = (1 - d)r_\theta - b^{\mu}_{\theta} - t|x_\theta - x|, \tag{23}$$

where the parameter d now interacts with an advertiser's returns. That is, new data restrictions entail a worsening in each ad network's ability to match an advertiser with its targeted users, which diminishes the advertiser's returns. In the proceeding, we make the base model's assumptions that d proportionally impacts advertisers' returns from the ad networks, and that

[19] See, e.g., https://support.google.com/google-ads/answer/2472742?hl=en.

the larger advertiser market has Lebesgue measure zero—the analyses of the preceding sections showed that these assumptions maintain the qualitative nature of the results. For ease of reference, we also maintain analogous or identical notation to the model in Section 5 where possible, and incorporate the participation constraint for large advertisers; that is, the advertising costs for large advertisers are bound above by the terms they can receive in the automated market.

Proposition 8 *New data restrictions have the following effects: (i) The share of advertisers that contract with the L (H) network as well as network L's (H's) profit increase (decrease). (ii) The cost of advertising rises on the L network and decreases on the H network. (iii) All advertisers' profits decrease.*

The findings in Proposition 8 indicate that advertisers, large and small, incur profit losses as a result of new data restrictions. These effects are summarized in Table 3.

Network	*H*	*L*
Market Share	Decreases	Increases
Advertising Cost	Decreases	Increases
Profit	Decreases	Increases
Advertiser	*Small*	*Large*
Profit	Decreases	Decreases

Table 3: Summary of the effects of new data restrictions on networks and advertisers

We next assess the relative effects on advertisers' profits by calculating a unitless elasticity measure. In an analogous manner to the model with publishers, this measure captures the percentage decline in advertiser profit that results from a percentage (proportional) decline in ad returns due to new data restrictions. Doing so makes it possible to identify where, ceteris paribus, the brunt of the negative effects among advertisers are incurred. In the following proposition, we consider a threshold $\widehat{x}_{IC}^{n,*}$ for large advertisers that is analogous to (and with the same notation as) the threshold for large publishers in the negotiated market, i.e., it identifies the subset $[\widehat{x}_{IC}^{n,*}, 1]$ of large advertisers for which the participation constraint in the negotiated market binds.

Proposition 9 *New data restrictions have the following effects:*

1. *The percentage profit declines that small advertisers incur are greater than or equal to those incurred by similarly positioned large advertisers, and strictly so for $x < \widehat{x}_{IC}^{n,*}$.*

2. *Small advertisers who contract with network L are harmed more than analogously positioned small advertisers who contract with H; the reverse holds for large advertisers.*

Proposition 9 extends the model's main results for publishers in Proposition 4 to the advertisers side of the overarching market. The findings demonstrate that small advertisers take the brunt of the impact of new data restrictions. Moreover, small advertisers who advertise on the lower-revenue network L are harmed more than their analogous counterparts who advertise on the higher-revenue network H. The results from Proposition 9 are summarized in Table 4.

Market Segment	*Small Advertiser*	*Large Advertiser*
Contract with L	More Harm	Less Harm
Contract with H, $\widehat{x}_{IC}^{n,*}$	More Harm	Less Harm
Contract with H, $x \geq \widehat{x}_{IC}^{n,*}$	Same Harm	Same Harm
Contract With Network	*L*	*H*
Small Advertiser	More Harm	Less Harm
Large Advertiser	Less Harm	More Harm

Table 4: Relative impact of new data restrictions on advertisers' profits

Combined, Propositions 4 and 9 paint a stark picture that it is those smaller market actors, both publishers and advertisers, and particularly those publishers and advertisers who contract with the lower-revenue ad network, who form the front line of profit casualties after new data restrictions are imposed.

9 Extensions

9.1 Endogenizing Publisher Revenue

Our framework examined the publisher and advertiser sides of the overarching market separately. In practice, an advertiser may seek to advertise to multiple publishers and contract with a separate buy-side network; publishers may seek to sell ad space to multiple advertisers and contract with a separate sell-side network; and competitive pressures, potential market entry, and market standards may all force some additional separation between the publisher and advertiser sides. In other words, from the perspective of ad networks, there may exist barriers to altering terms for one side (e.g., publishers) due to changes in the other (e.g., advertisers)–this is in fact evident in the terms offered by separate sell-side and buy-side ad networks–though some spillovers could and are likely to take place, though perhaps not in the short term. Although we have not endogenized the relationship between publishers and advertisers, doing so is possible and maintains the qualitative nature of the results.

Specifically, consider L and H networks that offer both sell-side and buy-side services to publishers and advertisers, respectively, in unified markets. If the revenues the ad networks can generate for publishers are monotonically related to the aggregate advertising payments the networks can attract from advertisers, which seems rather plausible, then the networks' profit objectives amount to jointly maximizing advertisers' payments and publishers' revenue-sharing terms. Such a joint maximization problem means that new data restrictions that degrade revenues for advertisers, as captured by a rise in d, entail that networks equate their marginal losses on the publisher side (by adjusting revenue-sharing terms) to their marginal losses on the advertiser side (by adjusting advertisers' payments). Provided that a network's objective function exhibits some degree of continuity over the relevant parameter ranges, then its objective necessitates that, in net, it will incur losses on both its advertiser and publisher sides. Hence, a spillover from the advertiser side to the publisher side would take place, with losses for both sides after a rise in d. It thus follows that the qualitative nature of the results obtained for sell-side and buy-side networks would continue to hold.

9.2 Heterogeneous Revenues

Throughout the preceding sections, we have simplified the analysis by assuming that the revenues that large publishers and advertisers can obtain per unit of ad space are equal to those that can be obtained by small publishers and advertisers, that is, that R_θ is the same for publishers and r_θ is the same advertisers, independent of their sizes. In practice, however, large advertisers may pursue more expensive types of advertising campaigns because they anticipate greater revenues, and large publishers' web content may attract significantly more traffic. Relaxing this assumption, however, only strengthens our results. More specifically, let $R_\theta^n > R_\theta^a = R_\theta$ and $r_\theta^n > r_\theta^a = r_\theta$ denote the revenues of large publishers and advertisers, respectively, i.e., the revenues of large publishers and advertisers are strictly greater than their small counterparts.[20]

[20]In terms of the preceding analysis, the only results that are affected by this change are the comparisons of the elasticities of profits with respect to a degradation in revenues after new data restrictions are imposed

Proposition 10 *Given $R^n_\theta > R_\theta$ and $r^n_\theta > r_\theta$, the percentage profit declines that small publishers and small advertisers incur are strictly greater than those incurred by their similarly positioned large counterparts for all $x \in [0, 1]$.*

Intuitively, the introduction of greater revenues for large publishers and advertisers scales up their profits. If one envisions those large publishers and advertisers' profits as a function of d, this scaling up results in some upward rotation (the horizontal intercept is marginally changed, whereas the vertical intercept rises). Consequently, their profit functions as a function of d become steeper, resulting in lower magnitude profit elasticities following a comparable change in d.[21] It follows that relaxing the assumption that large and small publishers and advertisers derive equal revenues from the networks in fact strengthens the results. Said another way, the preceding sections examine a more conservative case where $R^n_\theta = R_\theta$ and $r^n_\theta = r_\theta$; once we allow for revenues to increase in publishers' and advertisers' sizes, the results hold more strongly, as summarized in Table 5.

Market Segment	*Small Publishers*	*Large Publishers*	*Small Advertisers*	*Large Advertisers*
Contract with L	More Harm	Less Harm	More Harm	Less Harm
Contract with H	More Harm	Less Harm	More Harm	Less Harm

	Publishers		Advertisers	
Contract With	L	H	L	H
Small Size	More Harm	Less Harm	More Harm	Less Harm
Large Size	Less Harm	More Harm	Less Harm	More Harm

Table 5: Relative impact of new data restrictions on publishers' and advertisers' profits

10 Conclusion

We presented a framework comprising of publishers and two asymmetric ad networks, and extended it to consider advertisers. Publishers and advertisers in our framework differ along two dimensions: size and their preferences for contracting with the two sell-side and buy-side ad networks. The networks differ in terms of the revenues they can generate for publishers and advertisers. Using this framework, we examined the effects of a regulatory change concerning

(i.e., after a rise in d), for small and large publishers and advertisers. The results of these comparisons are in Proposition 4 (part 1) and Proposition 9 (parts 1 and 2).

[21] While we have not changed the corresponding preference cost t for large publishers and advertisers, provided Assumptions 1 and 2 continue to hold, doing so does not alter the results.

data usage that degrades advertising revenues. We demonstrated that our analysis is robust to the regulatory change having differential effects on the ad revenues that the two networks can generate, as well as to varying the relative sizes of the small and large publisher markets.

Legislative bodies around the world are considering imposing or have imposed new restrictions on data uses for marketing purposes. However, there is a lack of clarity about who stands to gain and who stands to lose from such restrictions. To the extent that such restrictions hamper ad networks' abilities to match users with ads and generate revenues for publishers and advertisers, our findings show that small publishers and small advertisers are at the forefront in terms of subsequent percentage profit losses, and particularly those that contract with smaller ad networks. Our findings further demonstrate that a relative increase in the size of the market for large publishers, due to, for instance, policies that encourage publisher alliances, may have further detrimental effects on small publishers.

Importantly, we derive all of our findings in the context of simple changes in the advertising revenues networks can deliver after new data restrictions are imposed, without incorporating other potential changes, such as liability changes in the advertising-related data supply chain. Such considerations may in fact further strengthen our results if weaker ad networks are more susceptible to raising data liability costs for publishers, because they themselves are less equipped to ensure compliance with new regulations.

Future work can more formally examine the relationship between sell-side and buy-side ad networks, and thus between the payments that advertisers pay to the buy-side networks and the ad revenues that the sell-side networks generate for publishers, as well as the arrangements between the sell-side and buy-side networks themselves. This is of particular interest because the dynamics that arise among ad networks may help shed more light on how the burdens of lower revenues due to new data restrictions are distributed to publishers and advertisers, factoring in competitive considerations from rival networks.

References

Acquisti, A., C. Taylor, and L. Wagman (2016): "The Economics of Privacy," *Journal of Economic Literature*, 54, 442–492.

ADACHI, T. AND M. J. TREMBLAY (2020): "Business-to-Business Bargaining in Two-Sided Markets," *European Economic Review*, 130, 103591.

ADJERID, I. AND M. G. DE MATOS (2019): "Behavior and Firm Targeting after GDPR: The Case of a Telecom Provider in Europe," Working Paper.

BERGEMANN, D. AND A. BONATTI (2015): "Selling Cookies," *American Economic Journal: Microeconomics*, 7, 259–294.

CAMPBELL, J., A. GOLDFARB, AND C. TUCKER (2015): "Privacy Regulation and Market Structure," *Journal of Economics & Management Strategy*, 24, 47–73.

CONITZER, V., C. TAYLOR, AND L. WAGMAN (2012): "Hide and Seek: Costly Consumer Privacy in a Market with Repeat Purchases," *Marketing Science*, 31, 277–292.

COOPER, J., L. FROEB, AND D. O'BRIEN (2005): "Does Price Discrimination Intensify Competition? Implications for Antitrust," *Antitrust Law Journal*, 72, 327–373.

DAVIS, W. (2020): "IAB France, Others Bring Antitrust Complaint In EU Against Apple Over Privacy," October 28, 2020, `https://www.mediapost.com/publications/article/357290/iab-france-others-bring-antitrust-complaint-in-eu.html`, Accessed Feb. 1, 2021.

DE CORNIÈRE, A. (2016): "Search Advertising," *American Economic Journal: Microeconomics*, 8, 156–88.

DE CORNIÈRE, A. AND R. DE NIJS (2016): "Online Advertising and Privacy," *The RAND Journal of Economics*, 47, 48–72.

DE CORNIÈRE, A. AND G. TAYLOR (2020): "Data and Competition: A General Framework with Applications to Mergers, Market Structure, and Privacy Policy," Working Paper.

D'ANNUNZIO, A. AND A. RUSSO (2019): "Ad Networks and Consumer Tracking," *Management Science*.

FUDENBERG, D. AND J. TIROLE (2000): "Customer Poaching and Brand Switching," *RAND Journal of Economics*, 31, 634–657.

GOLDBERG, S., G. JOHNSON, AND S. SHRIVER (2019): "Regulating Privacy Online: The Early Impact of the GDPR on European Web Traffic & E-Commerce Outcomes," Working Paper.

GOLDFARB, A. AND C. TUCKER (2011): "Privacy Regulation and Online Advertising," *Management Science*, 57, 57–71.

Hsiao, S. (2020): "How Our Display Buying Platforms Share Revenue with Publishers," June 23, 2020, https://blog.google/products/admanager/display-buying-share-revenue-publishers, Accessed Feb. 1, 2021.

Jia, J., G. Jin, and L. Wagman (2019): "The Short-Run Effects of GDPR on Technology Venture Investment," *Marketing Science*, forthcoming.

Jin, G. Z. and L. Wagman (2020): "Big Data at the Crossroads of Antitrust and Consumer Protection," *Information Economics and Policy*, 100865.

Johnson, G., S. Shriver, and S. Goldberg (2019): "Privacy & Market Concentration: Intended & Unintended Consequences of the GDPR," Working Paper.

Johnson, G. A., S. K. Shriver, and S. Du (2020): "Consumer Privacy Choice in Online Advertising: Who Opts out and at What Cost to Industry?" *Marketing Science*, 39, 33–51.

Kim, J.-H. and L. Wagman (2015): "Screening Incentives and Privacy Protection in Financial Markets: A Theoretical and Empirical Analysis," *RAND Journal of Economics*, 46, 1–22.

Kim, J.-H., L. Wagman, and A. Wickelgren (2019): "The Impact of Access to Consumer Data on the Competitive Effects of Horizontal Mergers and Exclusive Dealing," *Journal of Economics & Management Strategy*, 28, 373–391.

Kirpalani, R. and T. Philippon (2020): "Data Sharing and Market Power with Two-Sided Platforms," Tech. rep., National Bureau of Economic Research.

Libert, T., L. Graves, and R. K. Nielsen (2018): "Changes in Third-Party Content on European News Websites after GDPR," *Reuters Institute for the Study of Journalism*.

Marotta, V., V. Abhishek, and A. Acquisti (2019): "Online Tracking and Publishers' Revenues: An Empirical Analysis," in *Proceedings of the 18th Annual Workshop on the Economics of Information Security*.

Spangler, T. (2013): "YouTube Standardizes Ad-Revenue Split for All Partners, But Offers Upside Potential," November 1, 2013, https://bit.ly/2LhXN4Q, Accessed Feb. 1, 2021.

Stewart, B. (2020): "A Look at How News Publishers Make Money with Ad Manager," June 23, 2020, https://blog.google/products/admanager/news-publishers-make-money-ad-manager, Accessed Feb. 1, 2021.

Taylor, C. and L. Wagman (2014): "Consumer Privacy in Oligopolistic Markets: Winners, Losers, and Welfare," *International Journal of Industrial Organization*, 34, 80–84.

Thisse, J.-F. and X. Vives (1988): "On the Strategic Choice of Spatial Price Policy," *The American Economic Review*, 122–137.

Wilander, J. (2017): "Intelligent Tracking Prevention," June 5, 2017, `https://webkit.org/blog/7675/intelligent-tracking-prevention/`, Accessed Feb. 1, 2021.

Zawadzinski, M. (2018): "What Is an Ad Network and How Does It Work?" March 7, 2018, `https://clearcode.cc/blog/what-is-an-ad-network-and-how-does-it-work/`, Accessed Feb. 1, 2021.

Appendix

Proof of Proposition 1: We prove, in order, each of the three claims in the proposition.

Proof of Claim 1: First consider publishers $x \in [0, \tilde{x}^{a,*})$. As shown in Section 4.1, these publishers find it optimal to contract with L network and they receive the terms provided in (7). Differentiating (7) with respect to d yields $\frac{-t}{(1-d)^2 R_L}$, which is always negative. Similarly, differentiating $\Pi_P^*(x; a)$ given in (9), it is straightforward to see that publishers' profits are decreasing in d. The proof for publishers $x \in [\tilde{x}^{a,*}, 1]$ can be completed in an analogous manner.

Proof of Claim 2: First consider network $\theta = L$. As shown in Section (4.1), $M(L, a) = \{x | x \in [0, \tilde{x}^{a,*})\}$ where $\tilde{x}^{a,*}$ is given by (10). The first part of the claim follows immediately from the derivative of (10) with respect to d is positive. In order to establish the second part of the claim, we differentiate $\Pi_N^*(L, a)$ given by (11) with respect to d which yields,

$$\frac{\partial \Pi_N^*(L, a)}{\partial d} = \frac{(3t - (1-d)(R_H - R_L))(R_H - R_L)}{9t}.$$

Using Assumption 1, it can then be verified that $3t - (1-d)(R_H - R_L)$ is always positive. As $R_H - R_L$ is also positive, it can seen that $\Pi_N^*(L, a)$ is increasing in d.The claim related to network H can be completed in an analogous manner.

Proof of Proposition 2: We prove, in order, each of the three claims in the proposition.

Proof of Claim 1: First consider publishers $x \in [0, \tilde{x}^{n,*})$. As shown in Section 4.2, these publishers receive $a_L^{n,*}(x)$ given in (14). Differentiating (14) with respect to d yields:

$$\frac{\partial \alpha_L^{n,*}(x)}{\partial d} = \frac{-t(1-2x)}{(1-d)^2 R_L}.$$

For all $x < \tilde{x}^{n,*} < \frac{1}{2}$, $(1-2x)$ is always positive meaning that $\alpha_L^{n,*}(x)$ is decreasing in d for this range of publishers. Next, consider publishers $x \in [\tilde{x}^{n,*}, 1]$ that receive $a_H^{n,*}(x)$ given in (15). Differentiating (15) with respect to d yields:

$$\frac{\partial \alpha_H^{n,*}(x)}{\partial d} = \frac{t(1-2x)}{(1-d)^2 R_H}.$$

As can be seen from the above expression, the behavior of $\alpha_H^{n,*}(x)$ is determined by the sign of $1 - 2x$. The proof is completed by observing that $1 - 2x$ is positive for $\tilde{x}^{n,*} \leq x \leq \frac{1}{2}$, and is negative for $x > \frac{1}{2}$.

Proof of Claim 2: This result follows immediately by noticing the sign of the derivative of $\Pi_P^*(x; n)$, given by (19), with respect to d is negative for all publishers $x \in [0, 1]$.

Proof of Claim 3: First consider network $\theta = L$. As shown in Section 4.2, $M(L, n) = \{x | x \in [0, \tilde{x}^{n,*})\}$ where $\tilde{x}^{n,*}$ is given by (13). The first part of the claim follows immediately from the derivative of (13) with respect to d that is positive. In order to establish the second part of the claim, we differentiate $\Pi_N^*(L, n)$ given by (20) with respect to d which yields,

$$\frac{\partial \Pi_N^*(L,n)}{\partial d} = \frac{(t-(1-d)(R_H-R_L))(R_H-R_L)}{2t}.$$

From Assumption 1, it follows that $t > R_H - R_L > (1-d)(R_H - R_L) > 0$ establishing that $\Pi_N^*(L,n)$ is increasing in d. The claim related to network H can be completed in an analogous manner.

Proof of Proposition 3: We prove, in order, each of the four claims in the proposition.

Proof of Claim 1: As shown in Section 4.1 and 4.2, the share of publishers that contract with network L in market $\mu \in \{a,n\}$ is given by: $M(L,\mu) = \{x | x \in [0, \tilde{x}^{\mu,*})\}$ where $\tilde{x}^{a,*}$ is given by (10) and $\tilde{x}^{n,*}$ is given by (13). A direct comparison between (10) and (13) establishes that $\tilde{x}^{n,*} < \tilde{x}^{a,*}$.

Proof of Claim 2: First consider publishers $x \in [0, \tilde{x}^{n,*})$. The equilibrium revenue-share of these publishers in the two markets are given by (7) and (14). Subtracting (14) from (7), it can be seen that

$$\alpha_L^{a,*}(x) - \alpha_L^{n,*}(x) = -\frac{2(1-d)(R_H-R_L)+6tx}{3(1-d)R_L}$$

is negative, thereby establishing that $\alpha_L^{n,*}(x) > \alpha_L^{a,*}(x)$ for all publishers $x \in [0, \tilde{x}^{a,*})$. Next, we consider publishers $x \in [\tilde{x}^{n,*}, \tilde{x}^{a,*})$. The equilibrium revenue-share of these publishers in the two markets are given by (8) and (14). By proceeding in a analogous manner as for publishers $x \in [0, \tilde{x}^{a,*})$, it can be established that $\alpha_L^{n,*}(x) > \alpha_L^{a,*}(x)$ for all publishers $x \in [\tilde{x}^{a,*}, \tilde{x}^{n,*})$. The result then following by noting that $\alpha_L^{a,*}(x) > \alpha_H^{a,*}(x)$ for all $x \in [0,1]$. Finally, we consider publishers $x \in [\tilde{x}^{n,*}, 1]$. The equilibrium revenue-share of these publishers in the two markets are given by (8) and (15). Subtracting (15) from (8), it can be seen that

$$\alpha_H^{a,*}(x) - \alpha_H^{n,*}(x) = \frac{2(1-d)(R_H-R_L)-6t(1-x)}{3(1-d)R_H}$$

As can be seen from the above expression, the sign of $\alpha_H^{a,*}(x) - \alpha_H^{n,*}(x)$ depends on the sign of $(1-d)(R_H-R_L) - 3t(1-x)$. Through some simple algebraic manipulations, it can be established that $(1-d)(R_H-R_L) - 3t(1-\tilde{x}^{n,*})$ is negative, implying that $\alpha_H^{a,*}(x) - \alpha_H^{n,*}(x)$ is negative for $x = \tilde{x}^{n,*}$. It can also be verified by that $\alpha_H^{a,*}(x) - \alpha_H^{n,*}(x)$ is positive for $x = 1$. Finally, as $\alpha_H^{a,*}(x) - \alpha_H^{n,*}(x)$ is monotonically increasing in x, an application of Intermediate Value Theorem establishes the existence of $\tilde{x}_{IC}^{n,*}$ that satisfies $(1-d)(R_H-R_L) - 3t(1-x) = 0$.

The statement about publishers' profits follows from noting that $\Pi_P^*(x;\mu) = \alpha_\theta^{\mu,*}(x)(1-d)R_\theta$ where $\theta \in \{L,H\}$ denotes the equilibrium network choice of publishers.

Proof of Claim 3: Subtracting (19) from (11) after equalizing their denominators, we get $\Pi_N^*(L,a) - \Pi_N^*(L,n)$ as

$$\frac{(3\sqrt{2}t - \sqrt{2}(1-d)(R_H-R_L))^2 - (3t-3(1-d)(R_H-R_L))^2}{36t}$$

Define $a \equiv 3\sqrt{2}t - \sqrt{2}(1-d)(R_H - R_L)$ and $b \equiv 3t - 3(1-d)(R_H - R_L)$. As a and b are both positive, using the difference of squares formula, $a^2 - b^2 = (a-b)(a+b)$, it can be seen that the sign of $\Pi_N^*(L,a) - \Pi_N^*(L,n)$ is dependent on the sign of $a - b = 3(\sqrt{2}-1)t - (\sqrt{2}-3)(1-d)(R_H - R_L)$. As $1 < \sqrt{2} < 3$, it immediately follows that $a-b$ and in turn $\Pi_N^*(L,a) - \Pi_N^*(L,n)$ is always positive.

Proof of Claim 4: Subtracting (19) from (11) after equalizing their denominators, we get $\Pi_N^*(H,a) - \Pi_N^*(H,n)$ as

$$\frac{(3\sqrt{2}t + \sqrt{2}(1-d)(R_H - R_L))^2 - (3t + 3(1-d)(R_H - R_L))^2}{36t}$$

Define $a \equiv 3\sqrt{2}t + \sqrt{2}(1-d)(R_H - R_L)$ and $b \equiv 3t + 3(1-d)(R_H - R_L)$. As a and b are both positive, using the difference of squares formula, $a^2 - b^2 = (a-b)(a+b)$, it can be seen that the sign of $\Pi_N^*(H,a) - \Pi_N^*(H,n)$ is dependent on the sign of $a - b = 3(\sqrt{2}-1)t + (\sqrt{2}-3)(1-d)(R_H - R_L)$.Therefore, $\Pi_N^*(H,a) - \Pi_N^*(H,n)$ is positive whenever

$$t > \frac{3-\sqrt{2}}{3(\sqrt{2}-1)}(1-d)(R_H - R_L) \approx 1.27(1-d)(R_H - R_L).$$

Proof of Corollary 1: The result follows directly by combining the findings of Propositions 1 and 2.

Proof of Proposition 4: We prove, in order, each of the claims in the proposition.

Proof of Claim 1: First consider publishers $x \in [0, \tilde{x}^{n,*})$. Differentiating (9) with respect to d, we get

$$\frac{\partial \Pi_P^*(x;a)}{\partial d} = -\frac{\Pi_P^*(x;a) + t(1+x)}{1-d}$$

Similarly, differentiating (19) with respect to d, we get

$$\frac{\partial \Pi_P^*(x;n)}{\partial d} = -\frac{\Pi_P^*(x;n) + t(1-x)}{1-d}$$

Plugging the above expressions in (21), it can be shown that:

$$\frac{\epsilon_{\pi,d}(x;a)}{\epsilon_{\pi,d}(x;n)} = \frac{1 + \frac{t(1+x)}{\Pi_P^*(x;a)}}{1 + \frac{t(1-x)}{\Pi_P^*(x;n)}}.$$

It is obvious that $t(1-x) < t(1+x)$. Combining this with the finding in Proposition 3 that $\Pi_P^*(x;n) > \Pi_P^*(x;a)$, it follows that $\frac{t(1-x)}{\Pi_P^*(x;n)} < \frac{t(1+x)}{\Pi_P^*(x;a)}$, or equivalently, $|\epsilon_{\pi,d}(x;n)| < |\epsilon_{\pi,d}(x;a)|$. Next consider publishers $x \in [\tilde{x}^{n,*}, \tilde{x}^{a,*})$. By proceeding in a similar manner as above, it can be shown that

$$\frac{\epsilon_{\pi,d}(x;a)}{\epsilon_{\pi,d}(x;n)} = \frac{1+\frac{t(1+x)}{\Pi_P^*(x;a)}}{1+\frac{tx}{\Pi_P^*(x;n)}}$$

for publishers in this region. The result then follows by noting that $\frac{tx}{\Pi_P^*(x;n)} < \frac{t(1+x)}{\Pi_P^*(x;a)}$. Next, consider publishers $x \in [\tilde{x}^{a,*}, \tilde{x}_{IC}^{n,*})$. For these publishers it can be shown that

$$\frac{\epsilon_{\pi,d}(x;a)}{\epsilon_{\pi,d}(x;n)} = \frac{1+\frac{t(2-x)}{\Pi_P^*(x;a)}}{1+\frac{tx}{\Pi_P^*(x;n)}}.$$

The result then follows by noting that $\frac{tx}{\Pi_P^*(x;n)} < \frac{t(2-x)}{\Pi_P^*(x;a)}$. Finally consider publishers $x \in [\tilde{x}_{IC}^{n,*}, 1]$. As $\Pi_P^*(x;n) = \Pi_P^*(x;a)$ for these publishers (due to the participation constraint), it is immediate that $\frac{\epsilon_{\pi,d}(x;a)}{\epsilon_{\pi,d}(x;n)} = 1$.

Proof of Claim 2: Consider first the small publishers, i.e. when $\mu = a$. By proceeding in a manner identical to the proof of Claim 1, it can be shown that

$$\frac{\epsilon_{\pi,d}(x;a)}{\epsilon_{\pi,d}(1-x;a)} = \frac{1+\frac{t(1+x)}{\Pi_P^*(x;a)}}{1+\frac{t(1+x)}{\Pi_P^*(1-x;a)}}$$

The result then follows by noting that $\Pi_P^*(1-x;a) > \Pi_P^*(x;a)$ implying that the above ratio is always less than 1. Next consider the large publishers, i.e. when $\mu = n$. First consider publishers $x \in [0, \tilde{x}^{n,*})$ such that $1 - \tilde{x}^{n,*} > \tilde{x}_{IC}^{n,*}$. For such publishers

$$\frac{\epsilon_{\pi,d}(x;n)}{\epsilon_{\pi,d}(1-x;n)} = \frac{1+\frac{t(1-x)}{\Pi_P^*(x;a)}}{1+\frac{t(1+x)}{\Pi_P^*(1-x;a)}}.$$

As $t(1-x) < t(1+x)$ and $\Pi_P^*(x;a) > \Pi_P^*(1-x;a)$, this implies that $\frac{t(1-x)}{\Pi_P^*(x;a)} < \frac{t(1+x)}{\Pi_P^*(1-x;a)}$ and the above ratio is always less than 1. Finally consider large publishers $x \in [0, \tilde{x}^{n,*})$ such that $1 - \tilde{x}^{n,*} < \tilde{x}_{IC}^{n,*}$. For such publishers

$$\frac{\epsilon_{\pi,d}(x;n)}{\epsilon_{\pi,d}(1-x;n)} = \frac{1+\frac{t(1-x)}{\Pi_P^*(x;a)}}{1+\frac{t(1-x)}{\Pi_P^*(1-x;a)}}.$$

The result then follows by noting that $\Pi_P^*(1-x;a) > \Pi_P^*(x;a)$ implying that the above ratio is always less than 1. This completes the proof.

Proof of Proposition 5: Before proceeding to prove the claims made in Proposition 5, we first identify the equilibrium outcome in this market. Let α_θ^a denote the share of ad revenues offered by network θ to publishers in its automated market. By applying a similar reasoning as in Section 6, it can be shown that networks' profit-maximizing revenue-sharing terms in the automated market is given by:

$$\alpha_L^{a,*} = \frac{2}{3} + \frac{(1-\psi d)R_H - 3t}{3(1-d)R_L}; \text{ and} \quad \text{(A–1)}$$

$$\alpha_H^{a,*} = \frac{2}{3} + \frac{(1-d)R_L - 3t}{3(1-\psi d)R_H} \quad \text{(A–2)}$$

The share of publishers that contract with L and H network are given by $M(L,a) = \{x|x \in [0, \tilde{x}^{a,*})\}$ and $M(H,a) = \{x|x \in [\tilde{x}^{a,*}, 1]\}$ respectively, where

$$\tilde{x}^{a,*} = \frac{1}{2} - \frac{(1-d\psi)R_H - (1-d)R_L}{6t} \quad \text{(A–3)}$$

Small publishers' profits are given by:

$$\Pi_P^*(x;a) = \begin{cases} \frac{2(1-d)R_L + (1-\psi d)R_H}{3} - t(1+x) & \text{if} \quad 0 \le x < \tilde{x}^{a,*} \\ \frac{(1-d)R_L + 2(1-\psi d)R_H}{3} - (2-x)t & \text{if} \quad \tilde{x}^{a,*} \le x \le 1, \end{cases} \quad \text{(A–4)}$$

where $\tilde{x}^{a,*}$ is as given in (A–3). Finally, ad networks' profits are given by:

$$\Pi_N^{a,*}(\theta,a) = \begin{cases} \frac{(3t-(1-\psi d)R_H + (1-d)R_L)^2}{18t} & \text{if} \quad \theta = L \\ \frac{(3t+(1-\psi d)R_H - (1-d)R_L)^2}{18t} & \text{if} \quad \theta = H \end{cases} . \quad \text{(A–5)}$$

Equipped with this equilibrium characterization, we next prove the claims in Proposition 5.

Proof of Claim 1: First consider network $\theta = L$. As shown in Section (4.2), $M(L,a) = \{x|x \in [0, \tilde{x}^{a,*})\}$ where $\tilde{x}^{a,*}$ is given by (A–3). The first part of the claim follows immediately from noting that the derivative of (A–3) with respect to d is negative iff $\psi < \frac{R_L}{R_H}$ and is positive otherwise . In order to establish the second part of the claim, we differentiate $\Pi_N^*(L,n)$ given by (A–5) with respect to d, which yields:

$$\frac{\partial \Pi_N^*(L,a)}{\partial d} = \frac{(3t - (1-\psi d)R_H + (1-d)R_L)}{9t}(\psi R_H - R_L).$$

As $t > (1-\psi d)R_H - (1-d)R_L > 0$. Then $\Pi_N^*(L,a)$ is decreasing in d whenever $\psi < \frac{R_L}{R_H}$ and is increasing otherwise.The claim related to network H can be completed in an analogous manner.

Proof of Claim 2: First consider publishers $x \in [0, \tilde{x}^{n,*})$ who receive $a_L^{n,*}(x)$ given in (A–1). Differentiating (A–1) with respect to d yields

$$\frac{\partial \alpha_L^{a,*}}{\partial d} = \frac{(1-\psi)R_H - 3t}{3(1-d)^2 R_L}.$$

As can be seen from above expression, $\alpha_L^{a,*}$ is increasing in d if and only if $\psi < 1 - \frac{3t}{R_H}$ and is decreasing otherwise. Next, consider publishers $x \in [\tilde{x}^{n,*}, 1]$ who receive $a_H^{n,*}(x)$ given in (A–2).

Differentiating (A–2) with respect to d gives

$$\frac{\partial \alpha_H^{a,*}}{\partial d} = -\frac{(3t - R_L)\,\psi + R_L}{3(1-\psi d)^2 R_H}$$

It can be shown that $3t - R_L > 3R_H - 4R_L > 0$ where the first inequality follows from Assumption 1 and second inequality follows from Assumption 3. This proves that $\alpha_H^{a,*}(x)$ is always decreasing in d.

Proof of Claim 3: The result follows from the derivative of (A–4) with respect to d.

Proof of Proposition 6: Before proceeding to prove the claims made in Proposition 6, we first characterize the equilibrium outcome. By proceeding in a manner identical to Section 6, it can be shown that the marginal consumer who is indifferent between contracting with two networks is given by:

$$\tilde{x}^{n,*} = \frac{1}{2} - \frac{(1-\psi d)R_H - (1-d)R_L}{2t} < \tilde{x}^{a,*} \tag{A–6}$$

Further, the optimal revenue-share offered by network L is:

$$\alpha_L^{n,*}(x) = \begin{cases} \frac{(1-\psi d)R_H - t(1-2x)}{(1-d)R_L} & \text{if} \quad 0 \le x < \tilde{x}^{n,*} \\ 1 & \text{if} \quad \tilde{x}^{n,*} \le x \le 1 \end{cases}, \text{ and} \tag{A–7}$$

by network H is:

$$\alpha_H^{n,*}(x) = \begin{cases} 1 & \text{if} \quad 0 \le x < \tilde{x}^{n,*}, \\ \frac{(1-d)R_L + t(1-2x)}{(1-\psi d)R_H} & \text{if} \quad \tilde{x}^{n,*} \le x < \tilde{x}_{IC}^{n,*}, \\ \frac{2}{3} + \frac{(1-d)R_L - 3t}{3(1-\psi d)R_H} & \text{if} \quad \tilde{x}_{IC}^{n,*} \le x \le 1. \end{cases} \tag{A–8}$$

where $\tilde{x}^{n,*}$ is given in (A–6) and

$$\tilde{x}_{IC}^{n,*} = 1 - \frac{(1-\psi d)R_H - (1-d)R_L}{3t}. \tag{A–9}$$

These revenue-sharing terms yield the following profits for a large publisher located at $x \in [0,1]$:

$$\Pi_P^*(x;n) = \begin{cases} (1-\psi d)R_H - t(1-x) & \text{if} \quad 0 \le x < \tilde{x}^{n,*} \\ (1-d)R_L - tx & \text{if} \quad \tilde{x}^{n,*} \le x < \tilde{x}_{IC}^{n,*} \\ \frac{(1-d)R_L + 2(1-\psi d)R_H}{3} - (2-x)t & \text{if} \quad \tilde{x}_{IC}^{n,*} \le x \le 1 \end{cases}, \tag{A–10}$$

with the following profits for the two ad networks from large publishers:

$$\Pi_N^*(\theta, n) = \begin{cases} \frac{(t-((1-\psi d)R_H - (1-d)R_L))^2}{4t} & \text{if} \quad \theta = L, \\ \frac{((1-\psi d)R_H - (1-d)R_L + 3t)(5((1-\psi d)R_H - (1-d)R_L) + 3t)}{36t} & \text{if} \quad \theta = H. \end{cases} \tag{A–11}$$

Equipped with this characterization, the claims in Proposition 6 can be proved in a manner analogous to Proposition 5.

Proof of Proposition 7: Before proceeding to prove the claims made in Proposition 7, we first characterize the equilibrium in this modified framework. The characterization proceeds in a manner similar to Section 5. Given L network's optimization problem in (5), its best response function is given by:

$$\alpha_L^a\left(\alpha_H^a\right)=\frac{(1-d)\left(R_H\alpha_H^a+R_L\right)-t}{2(1-d)R_L}. \tag{A–12}$$

Similarly, H network's optimization problem in (22) yields the following best response function:

$$\alpha_H^a\left(\alpha_L^a\right)=\frac{(1-d)\left(R_L\left(\alpha_L^a+\gamma\right)+R_H\right)-t\left(1+\gamma\right)}{(\gamma+2)(1-d)R_H}. \tag{A–13}$$

Simultaneously solving (A–12) and (A–13) allows us to identify the optimal revenue-shares for L network as:

$$\alpha_L^{a,*}=\frac{2\gamma+2}{2\gamma+3}+\frac{(1-d)R_H-(2\gamma+3)t}{(2\gamma+3)(1-d)R_L}, \tag{A–14}$$

and those for H network as:

$$\alpha_H^{a,*}=\frac{2}{2\gamma+3}+\frac{(2\gamma+1)(1-d)R_L-(2\gamma+3)t}{(2\gamma+3)(1-d)R_H}. \tag{A–15}$$

Plugging (A–14) and (A–15) in (4) allows us to identify the marginal publisher as

$$\tilde{x}^{a,*}=\frac{1}{2}-\frac{(1-d)\left(R_H-R_L\right)}{2\left(2\gamma+3\right)t}. \tag{A–16}$$

Finally, using the condition that $\tilde{x}_{IC}^{n,*}$solves $\alpha_H^a=\alpha_H^{n,*}\left(x\right)$ where $\alpha_H^{n,*}\left(x\right)$ is given in (15), we get $\tilde{x}_{IC}^{n,*}$ as

$$\tilde{x}_{IC}^{n,*}=1-\frac{(1-d)\left(R_H-R_L\right)}{\left(2\gamma+3\right)t} \tag{A–17}$$

In conjunction with (1), and accounting for publishers' optimal choice of network, (A–14) and (A–15 can be used to derive the profits for small publishers in automated market as:

$$\Pi_P^*\left(x;a\right)=\begin{cases}\alpha_L^{a,*}\left(1-d\right)R_L-tx & \text{for}\quad 0\leq x<\tilde{x}^{a,*}\\ \alpha_H^{a,*}\left(1-d\right)R_H-t\left(1-x\right) & \text{for}\quad \tilde{x}^{a,*}\leq x\leq 1\end{cases}, \tag{A–18}$$

and networks' profits as

$$\Pi_N^*(\theta,a)=\begin{cases}\left(1-\alpha_L^{a,*}\right)\tilde{x}^{a,*}\left(1-d\right)R_L & \text{if}\quad \theta=L\\ \left(1-\alpha_H^{a,*}\right)\left(1-\tilde{x}^{a,*}\right)\left(1-d\right)R_H & \text{if}\quad \theta=H\end{cases}. \tag{A–19}$$

The revenue-sharing terms and profits of large publishers in negotatiated market can be derived in the same manner as in Section 5. In this case, the marginal publisher is still given by (13). $\alpha_L^{n,*}\left(x\right)$ coincides with (14) for all publishers. $\alpha_H^{n,*}\left(x\right)$ coincides with (15) for publishers $x\in[0,\tilde{x}_{IC}^{n,*})$. For publishers $x\in[\tilde{x}_{IC}^{n,*},1]$, $\alpha_H^{n,*}\left(x\right)=\alpha_H^{a,*}$ given in (A–15). Given $\alpha_L^{n,*}\left(x\right)$ and

$\alpha_H^{n,*}(x)$, publishers' profits are:

$$\Pi_P^*(x;n) = \begin{cases} (1-d)R_H - t(1-x) & \text{for} \quad 0 \le x < \tilde{x}^{n,*} \\ (1-d)R_L - tx & \text{for} \quad \tilde{x}^{n,*} \le x < \tilde{x}_{IC}^{n,*} \\ \left(1-\alpha_H^{a,*}\right)(1-d)\,R_H - t\,(1-x) & \text{for} \quad \tilde{x}_{IC}^{n,*} \le x \le 1 \end{cases}. \tag{A–20}$$

The corresponding profits for the networks in two markets are given by:

$$\Pi_N^*(\theta,n) = \begin{cases} \left(\tilde{x}^{n,*} - \int_0^{\tilde{x}^{n,*}} \alpha_L^{n,*}(x)\,dx\right)(1-d)\,R_L & \text{if} \quad \theta = L \\ \left(1 - \tilde{x}^{n,*} + \int_{\tilde{x}^{n,*}}^1 \alpha_H^{n,*}(x)\,dx\right)(1-d)\,R_H & \text{if} \quad \theta = H \end{cases} \tag{A–21}$$

Armed with this characterization, we next prove the claims in Proposition 7.

Proof of Claim 1: This claim follows immediately by differentiating (A–16) and (13) with respect to γ.

Proof of Claim 2: The claim relating to small publishers follows directly from differentiating (A–14), (A–15), and (A–18) with respect to γ. The claim relating to large publishers similarly follows from differentiating (14), (18), and (A–20) with respect to γ.

Proof of Proposition 8: We begin by characterizing the equilibrium. Given networks' costs b_θ^a, an individual advertiser's profit from contracting with network θ in market μ is given by (23). First, consider the automated market, $\mu = a$. For a given $b_\theta^a \in [0,1]$, it can be seen from (23) that $\Pi_A(L,a)$ is decreasing in x and $\Pi_A(H,a)$ is increasing in x. Therefore, there exists $\widehat{x}(b_H^a, b_L^a)$ such that all advertisers with $x < \widehat{x}(b_H^a, b_L^a)$ contract with network L and all advertisers with $x \ge \widehat{x}(b_H^a, b_L^a)$ contract with network H. $\widehat{x}(b_H^a, b_L^a)$ solves $\Pi_A(L,a) = \Pi_A(H,a)$ and is given by:

$$\widehat{x}(b_H^a, b_L^a) = \frac{1}{2} + \frac{b_H^a - b_L^a - (1-d)(r_H - r_L)}{2t}. \tag{A–22}$$

Network L then chooses b_L^a to maximize $b_L^a \widehat{x}(b_H^a, b_L^a)$ and network H chooses b_H^a to maximize $b_H^a(1 - \widehat{x}(b_H^a, b_L^a))$. The corresponding first-order conditions yield:

$$b_L^a(b_H^a) = \frac{1}{2}\left(t + b_H^a - (1-d)(r_H - r_L)\right); \qquad \text{and} \qquad b_H^a(b_L^a) = \frac{1}{2}\left(t + b_L^a + (1-d)(r_H - r_L)\right)$$

Simultaneously solving the above conditions, we get networks' optimal ad costs as

$$b_L^{a,*} = t - \frac{1}{3}(1-d)(r_H - r_L); \text{ and} \tag{A–23}$$

$$b_H^{a,*} = t + \frac{1}{3}(1-d)(r_H - r_L). \tag{A–24}$$

Plugging (A–23) and (A–24) in (A–22) gives

$$\widehat{x}_A^{a,*} = \frac{1}{2} - \frac{(1-d)(r_H - r_L)}{6t}. \tag{A–25}$$

Given (A–23), (A–24) and (A–25), Advertisers' equilibrium profits are given by:

$$\Pi_A^*(x;a) = \begin{cases} (1-d)\, r_L - b_L^{a,*} - tx & \text{for} \quad 0 < x < \widehat{x}_A^{a,*} \\ (1-d)\, r_H - b_H^{a,*} - t\,(1-x) & \text{for} \quad \widehat{x}_A^{a,*} \leq x \leq 1 \end{cases}, \tag{A–26}$$

and those of Networks' are given by:

$$\Pi_N^*(\theta, a) = \begin{cases} b_L^{a,*} \widehat{x}_A^{a,*} & \text{if} \quad \theta = L, \\ b_H^{a,*}\left(1 - \widehat{x}_A^{a,*}\right) & \text{if} \quad \theta = H. \end{cases} \tag{A–27}$$

Consider now the negotiated market, $\mu = n$. In this market, competition forces networks to reduce their offers $b_\theta^n(x)$ to every advertiser x. Suppose both networks offer $b_L^n(x) = b_H^n(x) = 0$. Then by the same reasoning as in the automated market, there exists $\widehat{x}^n$ such that advertisers with $x < \widehat{x}^n$ contract with network L and advertisers with $x \geq \widehat{x}^n$ contract with H.This $\widehat{x}^n = \widehat{x}(0,0)$ (given in (A–22)) and is given by

$$\widehat{x}^{n,*} = \frac{1}{2} - \frac{(1-d)\,(r_H - r_L)}{2t} \tag{A–28}$$

For each large advertiser $x < \widehat{x}^{n,*}$, network L sets $b_L^n(x)$ to make the advertiser indifferent about contracting with the rival network when $b_H^n(x) = 0$, which gives $b_L^n(x) = 0$ for all advertisers $\widehat{x}^{n,*} \leq x \leq 1$ and $b_L^n(x) = t\,(1-2x) - (1-d)\,(r_H - r_L)$ for advertisers $x \in [0, \widehat{x}^{n,*})$. Similarly, H network finds it optimal to choose $b_H^n(x) = 0$ for advertisers $x \in [0, \widehat{x}^{n,*})$ and $b_H^n(x) = (1-d)\,(r_H - r_L) - t\,(1-2x)$ for advertisers $\widehat{x}^{n,*} \leq x \leq 1$. The resulting profits for advertisers are given by:

$$\Pi_A^n(x) = \begin{cases} (1-d)\, r_H - t\,(1-x) & \text{for} \quad 0 < x < \widehat{x}^{n,*} \\ (1-d)\, r_L - tx & \text{for} \quad \widehat{x}^{n,*} \leq x \leq 1 \end{cases} \tag{A–29}$$

Comparing advertisers' profits in (A–26) and (A–29), it can be shown that that there exists $\widehat{x}_{IC}^{n,*} > \widehat{x}^{n,*}$ such that for $x > \widehat{x}_{IC}^{n,*}$, large advertisers will have an incentive to switch over to the automated market. For such a switch not to take place, these publishers need to be offered terms $b_H^n(x)$ that equal $b_H^{a,*}$. Hence, the networks' optimal ask prices are set at:

$$b_L^{n,*}(x) = \begin{cases} t\,(1-2x) - (1-d)\,(r_H - r_L) & \text{for} \quad 0 \leq x < \widehat{x}^{n,*} \\ 0 & \text{for} \quad \widehat{x}^{n,*} \leq x \leq 1 \end{cases} \tag{A–30}$$

and

$$b_H^{n,*}(x) = \begin{cases} 0 & \text{for} \quad 0 \leq x < \widehat{x}^{n,*} \\ (1-d)\,(r_H - r_L) - t\,(1-2x) & \text{for} \quad \widehat{x}^{n,*} \leq x < \widehat{x}_{IC}^{n,*} \\ b_H^{a,*} & \text{for} \quad \widehat{x}_{IC}^{n,*} \leq x \leq 1, \end{cases} \tag{A–31}$$

where

$$\widehat{x}^{n,*} < \widehat{x}_{IC}^{n,*} \equiv 1 - \frac{(1-d)(r_H - r_L)}{3t}.$$

The corresponding profits for large advertisers are given by:

$$\Pi_A^*(x;n) = \begin{cases} (1-d)\,r_H \; - t\,(1-x) & \text{for} \quad 0 \le x < \widehat{x}^{n,*} \\ (1-d)\,r_L \; - tx & \text{for} \quad \widehat{x}^{n,*} \le x < \widehat{x}_{IC}^{n,*} \\ (1-d)\,r_H - b_H^{a,*} - t\,(1-x) & \text{for} \quad \widehat{x}_{IC}^{n,*} \le x \le 1, \end{cases}, \tag{A–32}$$

where $b_H^{a,*}$ are given in (A–24). Networks' profits are given by:

$$\Pi_N^*(\theta;n) = \begin{cases} \int_0^{\widehat{x}^{n,*}} b_L^{n,*}(x)dx & \text{if} \quad \theta = L, \\ \int_{\widehat{x}^{n,*}}^1 b_H^{n,*}(x)dx & \text{for} \quad \theta = H. \end{cases} \tag{A–33}$$

Armed with the preceding, we prove the claims in the proposition.

Proof of Claim 1: The first part of the claim follows from differentiating (A–25) and (A–28) with respect to d. The second part of the claim follows from differentiating (A–27) and (A–33) with respect to d.

Proof of Claim 2: This result follows from differentiating $b_\theta^{\mu,*}$, as given in (A–23), (A–24),(A–30), and (A–31) with respect to d.

Proof of Claim 3: This result follows from differentiating $\Pi_A^*(x;\mu)$ given in (A–26) and (A–32) with respect to d.

Proof of Proposition 9: We prove, in order, each of the claims in the proposition.

Proof of Claim 1: First consider advertisers $x \in [0, \widehat{x}^{n,*})$. Differentiating (A–26) with respect to d, we get

$$\frac{\partial \Pi_A^*(x;a)}{\partial d} = -\frac{\Pi_A^*(x;a) + b_L^{a,*} + tx}{1-d}.$$

Similarly, differentiating (A–32) with respect to d, we get

$$\frac{\partial \Pi_A^*\,(x;n)}{\partial d} = -\frac{\Pi_A^*\,(x;n) + t\,(1-x)}{1-d}$$

Plugging the above expressions in (21), it can be shown that:

$$\frac{\epsilon_{A,d}\,(x;a)}{\epsilon_{A,d}\,(x;n)} = \frac{1 + \frac{b_L^{a,*}+tx}{\Pi_A^*(x;a)}}{1 + \frac{t(1-x)}{\Pi_A^*(x;n)}}.$$

As $b_L^{a,*} + tx > t\,(1-x)$ and $\Pi_A^*\,(x;a) < \Pi_A^*\,(x;n)$, it follows that the above ratio is greater than 1. By proceeding in a similar manner, it can be verified that the above ratio exceeds 1 for all advertisers $x \in [\widehat{x}^{n,*}, 1]$.

Proof of Claim 2: Consider first the small advertisers $x \in [0, \widehat{x}^{n,*})$. By proceeding in a manner identical to the proof of Claim 1, it can be shown that

$$\frac{\epsilon_{A,d}\left(x;a\right)}{\epsilon_{A,d}\left(1-x;a\right)}=\frac{1+\frac{b_L^{a,*}+tx}{\Pi_A^*(x;a)}}{1+\frac{b_H^{a,*}+tx}{\Pi_A^*(1-x;a)}}.$$

As $b_L^{a,*} > b_H^{a,*}$ and $\Pi_A^*\left(x;a\right) < \Pi_A^*\left(1-x;a\right)$, it follows that the above ratio is greater than 1. The result related to large publishers can be proved in an analogous manner.

Proof of Proposition 10: The result relating to publishers can be established by proceeding in a manner identical to the proof of Claim 1 in Proposition 4. The result relating to advertisers can be established by proceeding in a manner identical to the proof of Claim 1 in Proposition 9.